

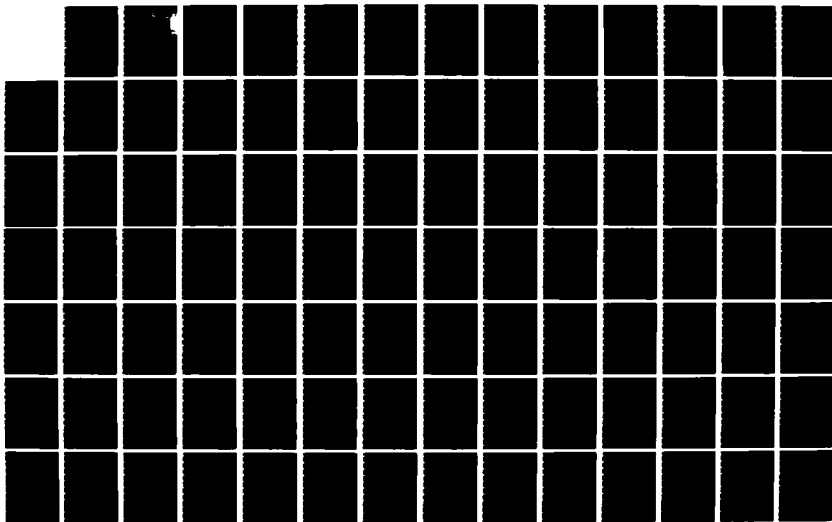
AD-A163 192

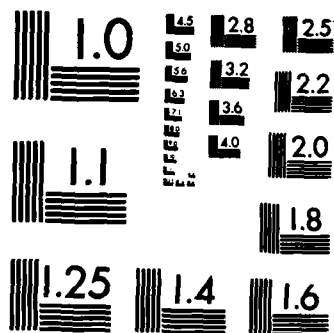
INTERACTIONS BETWEEN SIGNALING AND REPEATED PLAY WITH
BORROWER DEFAULT(U) STANFORD UNIV CA INST FOR
MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES C L SUCH
OCT 85 TR-480 N00014-79-C-0685 F/G 5/3

1/5

UNCLASSIFIED

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A163 192

INTERACTIONS BETWEEN SIGNALING AND REPEATED PLAY
WITH BORROWER DEFAULT

by

Carol L. Such

Technical Report No. 480
October 1985

THE ECONOMICS SERIES

INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES

FOURTH FLOOR, ENCINA HALL

STANFORD UNIVERSITY

STANFORD, CALIFORNIA

DTIC FILE COPY

This document has been approved
for public release and sale; its
distribution is unlimited.



1

INTERACTIONS BETWEEN SIGNALING AND REPEATED PLAY
WITH BORROWER DEFAULT

by

Carol L. Such

Technical Report No. 480
October 1985

A REPORT OF THE
CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY

Contract ONR-N00014-79-C-0685, United States Office of Naval Research

DTIC
SELECTED
JAN 21 1986
A

THE ECONOMICS SERIES
INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Fourth Floor, Encina Hall
Stanford University
Stanford, California
94305

This document has been approved
for public release and sale, its
distribution is unlimited.

(1)

INTERACTIONS BETWEEN SIGNALING AND REPEATED PLAY
WITH BORROWER DEFAULT*

by

Carol L. Such

1. Introduction

Much of the literature on credit and lending practices makes frequent but casual reference to the role of standing relationships between creditors and borrowers in common financial transactions. It is natural to expect that these relationships are valuable for both parties and that the actions of both the borrower and the lender reflect their worth. However, very little effort has been made to systematically investigate the benefits derived from maintenance of such a relationship.

Nearly all loan contracts involve some risk of default. The borrowing firm generally has more precise information than does the lender about the degree of risk involved in its particular operation and the business practices best suited to it. The borrower can sometimes exploit this informational asymmetry in the short run. In particular, a high-risk firm might try to convince the lender that it is, in fact, a good credit risk, thereby securing a lower interest rate. Such deception could subject the lender to significant probable loss while increasing the borrower's share of the potential gain.

*This research was supported by the Office of Naval Research Grant ONR-N00014-79-C-0685 at the Institute for Mathematical Studies in the Social Sciences, Stanford University.



Dist	Avail and/or Special
A-1	

Repeated transactions between the same lender and borrower might curtail some of this short-term opportunism. A borrower that defaulted would have difficulty recontracting a loan on favorable terms. To avoid this possibility, the borrower would be more likely to operate in a manner that served the interests of both the lender and itself. On the other hand, the lender might find it profitable to commit itself to repeated transactions at relatively lenient contract terms, conditional on the firm's cooperation. For example, a lender might promise a firm continued low interest rates on its loans if the firm maintains certain minimum performance criteria. Thus, both the lender and the borrower ultimately can benefit from the standing relationship.

The lender might also insist on exercising some degree of control over the borrowing firm in an effort to overcome the potential conflict of incentives of the two parties to the loan contract. It can induce compatible incentives by making a signal prerequisite to the extension of credit. The creditor has acquired experience from lending to other firms over the years and has come to associate certain business practices with creditworthy enterprises. It then monitors the business practices of its current loan customers by checking them against a generalized set of performance criteria based on past lending experience. The firm's choice of business practices becomes a signal of its creditworthiness. As in all signaling models, the signaling device is not costless. In this case, the firm might have to sacrifice some of its productive efficiency in order to meet the standards imposed by the lender.

This is a study of the signaling literature in the context of credit markets.

Signaling in credit markets has received considerable attention in the economics literature, but the signaling models involve only a single transaction.¹ Many credit models allude to repeated transactions. The few in which repetition is formally analyzed incorporate no other mechanism to overcome the incentive problem.² The models developed in this thesis will employ both. The primary aim is to study the effects of signaling in the context of standing relationships between the lender and borrowers.

The ability of the lender to make commitments to its loan customers is critical in determining the optimal loan contracts offered. These commitments might take the form of promising to grant favorable contract terms in return for the firm's cooperation. Alternatively, the lender might commit itself to punishing a firm that is suspected of having wittingly misrepresented itself. The lender's optimal decisions when it is free to make any form of commitment will be compared to the decisions that obtain when only restricted forms of commitments are allowed. This comparison is of interest because the ability to make commitments is related to the degree of ex post market power wielded by the creditor. If a lender is the sole source of funding for all of a firm's future borrowing, even extreme commitments can be enforced. However, if the borrower can freely break its ties with one creditor and seek alternative financing, each lender's ability to make credible commitments is severely impaired.

The firms in the model are small business entities in the early stages of development. All have production functions of the same form,

but the magnitudes of the production parameters depend upon which of two types of technology is available. A random component enters each production function; the distribution of the random variable is specific to the type of technology. Knowledge of the exact parameters cannot be observed readily; rather, this information requires intimate working knowledge of the specific firm's operations. Hence, a high-risk firm might profit from its informational advantage by purposely misrepresenting its technology to the lender in an effort to appear as if it were a low-risk firm.

The borrower's access to alternative sources of funding is crucial in delimiting the impact of lender commitments. The firm's behavior cannot be bound by anything if other resources are readily available. In order to emphasize the role of commitments, therefore, very restrictive assumptions are made about the financial arrangements available to the firms. Each relies solely on debt financing for its short-term input costs. Furthermore, no form of internal financing is available since these firms are only in the start-up stages of development. In other words, the loan from the lender is essential to each firm's operations.

The lender is allowed to offer loan contracts that specify not only the interest rate charged on borrowed funds, but also dictate parameters of the proposed business plan. Although this might seem an unrealistically extreme degree of managerial intervention, lending institutions often do set forth criteria that reflect their perception of sound business management practices. Failure to comply with these

standards can constitute grounds for the refusal of extensions of credit. The firm's agreement to the division of funds specified in any particular loan contract signals information to the lender about the firm's technology. The contract parameters might be set in such a way that only firms with one type of technology would accept a particular contract. Knowing that, the lender might be able to increase its profits by tailoring the loan contracts more nearly to the degree of risk involved.

A review of the literature relevant to the thesis is presented in the next section. This includes work on signaling and principal-agent models as well as a summary of the research on credit rationing. Following Section 2, two distinct models of standing relationships between borrowers and their creditor are analyzed in detail. In an effort to focus on other aspects of the negotiations between the contracting parties, credit rationing is explicitly ruled out of these models by specific assumptions about the technology available to each borrowing firm.

The model developed in Section 3 allows firms to differ in the size of the loan requested. If there is sufficient variation in size within each technology group, knowledge of the dollar amount requested reveals nothing to the lender about the borrower's true technological type. Although the number of each type of borrower in the market is fixed exogenously, the dollar amount requested by each borrower depends on the particular contract terms offered. The lender can determine the size of the loan demand as a function of the contract parameters.

Taking account of this dependence, the lender then selects terms to maximize its expected returns from all borrowers over the two-period horizon.

The loan size is held constant at the same level for all borrowers in the model presented in Section 7. However, there is a distribution of reservation wages among the entrepreneurs; hence, the aggregate demand for loans facing the lender increases as the contract parameters become more lucrative to the borrowing firms. In selecting these contract terms, the lender faces a trade-off between the expected return from a given loan and the number of loan applicants. Since a borrower is eligible for a loan in the second period only if he secured one in the initial round of contracting, the intertemporal dependence among contract parameters becomes quite complex.

Standing relationships between lenders and borrowers, coupled with managerial control on the part of the creditor, are phenomena observed in credit markets. The element of managerial control seems especially important when the borrowers are small, start-up firms. Venture capital markets provide prototypical examples of these practices. Many of the features of these markets, particularly the sharing of information, are strikingly suggestive of lenders' cartels. Although anticompetitive practices are normally considered inefficient, in this instance the market power of the cartel might help to alleviate the burden of the externality caused by the potentially dishonest firms in the market. In the model that follows, the value of the standing relationship turns on the lender's ability to make commitments to its borrowers. Lenders that

have this ability, of course, must also exercise some degree of market power. Market structures that are inefficient in complete information contexts might be less objectionable when informational asymmetries are inherent.

2. Literature Review

Initial attempts to analyze credit markets appeared in the economic journals in the early 1950's.^{3/} Researchers were puzzled by the phenomenon of credit rationing. Received theory predicted that in the face of excess demand, price would rise to clear the market. Yet in credit markets, the interest rate seemed to remain relatively stable even though borrowers were requesting more money than was available for lending. Instead, rationing in the form of smaller loans or complete refusal of funds was used to deal with the excess demand.

Interest in credit market transactions has remained active to the present. Increasingly sophisticated models have been built to study credit rationing as economists have come to view the problem as one of conflicting incentives between the borrower and the lender. The lender is more concerned with the downside risk of any investment project since repayment of the loan generally requires a prescribed minimum return. The borrower, on the other hand, is usually protected from severe loss by a limited liability clause. Moreover, he only gains from the investment when the returns are relatively high. Consequently, the borrower is often more willing to undertake high-risk projects financed by borrowed funds than is his creditor.

The problem of conflicting incentives is exacerbated by asymmetries of information common in most credit transactions. Since the borrower is the one who actually is involved with the investment project first hand, he typically has a more accurate sense of the true degree of risk borne by the contracting parties. There is no way to close the gap between the two, short of having the lender literally take over the business of the borrowing firm. The lender, being aware of its disadvantage, might insist on contract terms that force the borrower to reveal some of its private information.

The contract terms offered by a lender influence the actions of a potential borrower in at least two different ways. They are a primary determinant of each entrepreneur's decision to borrow. Unfavorable loan terms can leave the lender with only high-risk borrowers. The lower-risk entrepreneurs self-select out of the market because the expected return on their investment projects is likely to cover the debt to the lender but leave very little profit for the business. Moral hazard problems must also be recognized. Since there is generally a lower bound on the losses that can be incurred by the entrepreneur, a high interest rate on loanable funds can cause a borrower to choose relatively risky investment projects. When the borrower is covered by limited liability, the choice of a more risky project yields higher expected profit. The expected return from a successful outcome is more than adequate compensation for the increased probability of default.

Two basic approaches, signaling and principal-agent models, have been developed to analyze incentive problems. In order for signaling

mechanisms to be effective, there must be an inverse relationship between a particular characteristic of the economic agent and the cost of signaling its identity. When this condition is satisfied, the presence of this characteristic acts as a signal. For example, more credit-worthy borrowers need to sacrifice less than high-risk investors in order to meet the lender's criteria prerequisite to qualification for a low interest rate. Their willingness to cooperate signals their true risk category. If this inverse relationship fails to hold, the lender is precluded from accurately distinguishing borrowers and is forced to construct contracts knowing only the general composition of the pool of borrowers.

In principal-agent models, the principal might be incapable of distinguishing agents a priori; but it does know how each agent will react to its decisions, conditional on the agent's attributes. Therefore, it can anticipate these responses and choose its own actions accordingly. For example, the lender might be able to anticipate the effects of specific contract terms on the borrower's investment decisions. It might also be able to determine the type of borrower that would be attracted by each of several different contract offerings.

Concepts from both the signaling and the principal-agent literature are used extensively throughout the thesis. Some background of the development of each of these models will be provided; then applications specific to credit markets will be investigated.

2.1 Signaling Literature

Spence [1973] opened the door to a fascinating line of research with Job Market Signaling. Signaling models have since been used to address problems of adverse selection in labor markets, insurance markets, credit markets, and many others. In the simplest version of the Spence model, workers belong to one of two groups. They are observationally identical, but the marginal product of workers in one group is higher than that of the others. An employer can sort the two groups of workers by requiring a minimum level of education as prerequisite to a given wage offer. The cost of acquiring education diminishes with the worker's ability. Therefore, the employer can offer a high wage-high education package that is attractive only to the more productive workers. Such a wage schedule is termed "informationally consistent." That is, each worker's ability level is unambiguously identified by the wage-education combination he selects. Figure 1 illustrates a wage and education schedule that separates workers according to their marginal productivities.

This sorting of classes of workers promotes efficiency in production, particularly if the employer must assign workers to jobs requiring different levels of skill. However, the asymmetry in information, which causes the employer to resort to signaling devices, is costly in terms of social welfare. Education yields a higher personal gain than societal gain; consequently, more than the socially optimal level of investment in education is undertaken.

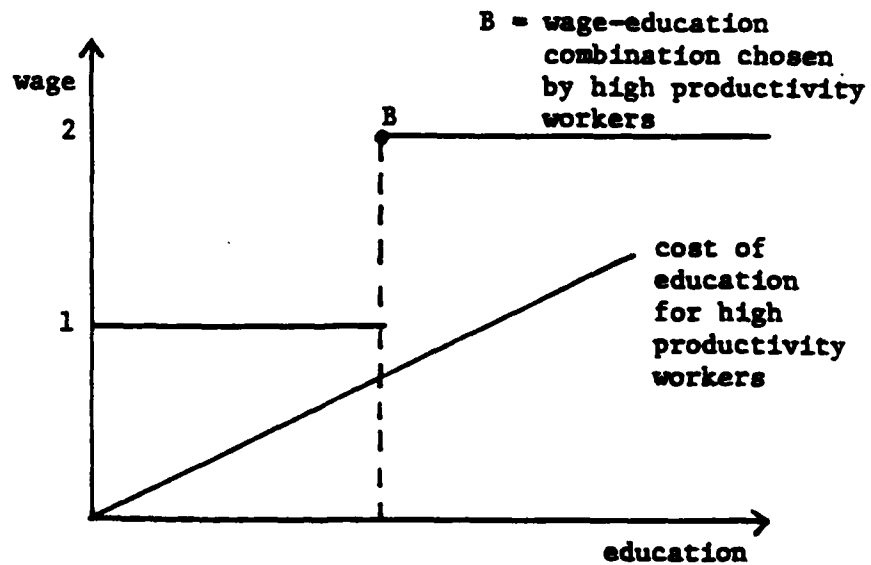
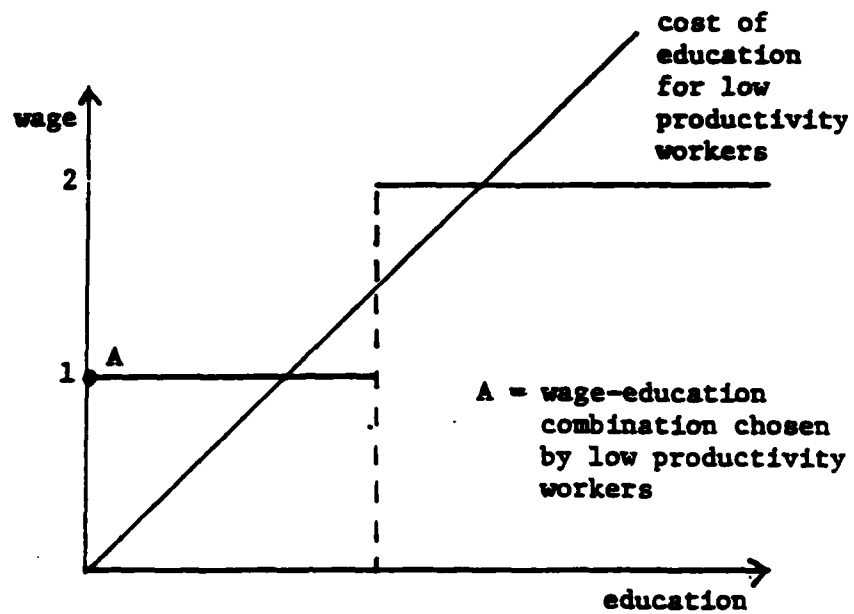


Figure 1: Education permits separation of workers on the basis of marginal productivity.

Wilson [1977] and Rothschild and Stiglitz [1976] simultaneously developed very similar models of signaling in the insurance market. In both models, consumers differ only in the probability of suffering an accident. The probability of this event is known to the consumer but not to the insurer. Hence, the insurer is prevented from simply charging different premia to different risk classes of consumers. Instead of offering unlimited insurance at a market-determined rate, insurance companies compensate for their informational disadvantage by offering price-quantity packages. Each consumer can then choose the premium-coverage combination commensurate with his known expectation of incurring an accident.

In the presence of informational asymmetries, high-risk consumers in the insurance market exert an externality on the low-risk purchasers. Actuarially fair contracts offering full insurance to low-risk people cannot be offered without attracting both high-risk and low-risk purchasers. The high-risk consumers cause expected losses from these policies. In some cases, this asymmetry can lead to complete market failure. When a set of informationally-consistent contracts does exist, the policies reflect this externality. The high-risk group is offered full insurance while the low-risk consumers are always underinsured.

In these models, markets are competitive in the sense that entry is free and no firm anticipates a reaction to any of its own changes in policy offerings. These conditions imply that a potential set of equilibrium contracts can be upset by any contract that is expected to be profitable, taking the existing contracts as given. It is not

surprising that equilibrium may not be sustainable under the assumptions of myopic rationality.

Riley [1979] demonstrated that in both the labor market and the insurance market, informationally consistent signaling equilibria can be upset by competitive employers that fully comprehend the operation of the signaling process. The purported labor market equilibrium can be unraveled with a new set of wage offers to a group of workers at the low end of the marginal productivity spectrum. These offers leave the workers at least as well off as they were originally but allow the employer to pay a wage rate that is less than the average marginal productivity of the group. A reinterpretation of symbols leads to the same conclusion in the insurance market. The nonexistence problem in competitive signaling models calls the predictive ability of this theory into question.

Two methods of restoring equilibrium in signaling models have been studied. Each approach endows the firms with a more sophisticated level of rationality than the models discussed above. This is justified when a typical firm has sufficient experience to learn to anticipate some reaction to each of its contract offerings. Wilson [1977] suggested a defensive sort of rationality. The introduction of a new contract into an initial set of offerings can render some of the existing contracts unprofitable. These unprofitable contracts are immediately withdrawn from the market. Since all firms can anticipate this reaction, no contract will be introduced unless it will make expected profits after the withdrawal of the unprofitable offerings. Riley [1979] proposed a

more aggressive rationality concept. No new contract can be introduced to upset the original set if it is possible for another firm to introduce a second new contract that undercuts the first.

When a sustainable set of contracts exists in the Rothschild-Stiglitz model, all three equilibrium concepts lead to the same result. Moreover, even when the market fails in the Rothschild-Stiglitz model, both the Wilson and the Riley alternative rationality assumptions ensure existence of an equilibrium set of contracts. The form of the contracts that obtain, however, is very sensitive to the precise specification of the firms' rationality. The Wilson equilibrium can involve pooling of heterogeneous types. In contrast, the Riley equilibrium selects the unique, Pareto-dominating member of the class of informationally consistent price schedules.

In some markets, particularly those in which the relationship between the buyer and seller is more tenuous than in the labor or insurance markets, there may be no incentive for an effective signal to emerge. If both the buyer and seller actively participate in the market only sporadically, there may be insufficient contact to learn which characteristics are positively correlated with quality and performance. In these markets, sellers typically know more about the product they are offering than do the buyers. The price at which goods are traded conveys some information to the buyers. However, a single price can be indicative of only the average quality of goods in the market; it cannot distinguish classes of quality.

Akerlof's [1970] famous "lemons" model was the first analysis of a market with asymmetric information but no means of accurately communicating information to the less well-informed agents. The used-car market was the prototypical example of such transactions. A seller knows the history of his used car, so he knows whether the car is good or a "lemon." A buyer, on the other hand, can only assess average quality of used cars by the market price. Consequently, sellers have an incentive to retain good cars and market only the poorer quality ones. A modified Gresham's law develops. A high-quality car cannot be sold for its true value; it will bring only the average market price. Consequently, the presence of the low-quality cars drives the high-quality ones out of the market. The difference between the private and social gains causes the quality and size of the market to diminish. In the most extreme case, the market can collapse completely.

Wilson [1980] extended the Akerlof used-car market model to investigate the dependence of the form of equilibrium on the convention used to set prices. The relative value attached to a car of a given quality is distributed continuously across the population. In this model, the demand curve is not necessarily downward-sloping since the price serves as an indicator of quality. As price rises, owners of better quality cars become willing to sell, thereby raising the average quality of used cars in the market. Depending on the distribution of car valuations, the demand curve can become even less steeply sloped than the supply curve over some range, resulting in multiple equilibria. Figure 2 exemplifies this phenomenon. Regardless of whether buyers or sellers



Figure 2: Multiple equilibria are possible when price reflects information about product quality.

set the price of the cars, the equilibrium need not be characterized by a single market-clearing price. Instead, a distribution of prices is possible. Moreover, excess demand can prevail at some or all of the prices.

Most of the models discussed thus far have the common characteristic of sellers possessing an informational advantage over buyers in the market. The presence of sellers of low-quality products inflicts an externality on those with high-quality goods. As a result, equilibrium may not exist. When an equilibrium is sustainable, the contract specifications take account of this externality. Despite the divergence between private and social gains, however, it need not be true that the asymmetry in information forestalls any benefit from trade.

Early models of credit markets implicitly assumed a structure much like the Spence labor market signaling model.^{4/} The interest rate charged to borrowers was analogous to the wage rate, and the size of the loan was used as the signal. This relationship was discussed at some length, but the signaling mechanism was never formally analyzed. Furthermore, the possibility of something other than size acting as the signal of creditworthiness was never entertained.

More recently, typical models of credit market transactions have recognized the complications of asymmetric information but have assumed that no effective method of distinguishing borrowers is feasible.^{5/} At best, the lender can only determine the average characteristics of the pool of loan applicants. Consequently, the lender offers contracts specifying only the interest rate; furthermore, the same interest rate

is offered to all loan applicants in the market. These models more closely parallel the Akerlof used-car model than the Spence labor market model. The strong restrictions on the lender's actions pave the way for credit rationing results. Lenders are unwilling to raise the interest rate, even in the face of excess demand, because a high interest rate adversely affects the "average riskiness" of the pool of actual borrowers.

In fact, lenders go to great lengths in an attempt to assess the creditworthiness of potential borrowers.^{6/} They strive to offer contracts that are molded to fit the needs of the customers but still provide adequate compensation for the risk assumed by the creditor. In reality, a number of instruments other than the interest rate are used to mediate the differences among borrowers. Credit histories, character references, and various financial ratios can play the same signaling role in credit markets as education does in the labor market. When the borrower is a new, risky business enterprise, direct involvement in the firm's basic operating procedures is often the most cost-effective signal of creditworthiness available to the lender. Because this evaluation process constitutes a major aspect of the creditor's role, some sense of the costs and benefits derived from it should be incorporated into economic models of lender behavior.

2.2 Principal-Agent Literature

Principal-agent models have been used extensively to study issues of optimal incentive schemes when moral hazard must be considered and to study optimal risk sharing when one or both parties are risk averse.

These particular issues are not directly relevant to the thesis research; however, an understanding of the structure of these models is useful. Attention will be focused on those principal-agent models that explicitly address repetition between the two parties. Aspects of these models are applicable to standing relationships between borrowers and creditors.

Rubenstein and Yaari [1980] considered an optimal punishment strategy for a price-setting property insurer when it is understood that the principal-agent relationship will continue indefinitely. The agent owns an asset which is subject to random risk of damage. This risk can be probabilistically reduced if the agent exercises proper care. However, the care is not costless. Once the principal sets the insurance premium, the agent must first decide whether to insure the asset at all; then the optimal level of care must be selected, given the choice of insurance. The principal can observe neither the actual level of care undertaken nor the random state of nature. Consequently, it cannot base the insurance on the agent's choice of care.

The agent in the model is risk averse, while the principal is risk neutral. Hence, full insurance for the agent provides optimal risk sharing; but it also raises a moral hazard problem. The agent is left with no incentive to spend anything on care. If a cooperative decision were possible, no inefficiency would arise. However, the conflict of interest between the two parties and the presence of informational asymmetries lead to some degree of inefficiency in a single transaction.

When both parties believe the relationship will be ongoing, the inefficiency due to moral hazard can be alleviated by charging premia based on an experience rating. If the number of claims submitted by the agent over a period of time is too large, the principal can conclude that the agent is not employing a sufficiently high level of care and can adjust the fee upward. Since the agent knows this in advance, it is induced to exercise the socially optimal amount of care in order to keep the premia at the lower level.

The principal's problem is one of finding a statistical technique that will simultaneously minimize the probability of two types of errors. The insurer's strategy should not punish the insured when he has, indeed, exercised the proper level of care but nature has inflicted damage anyway; nor should it be so lenient that the insured can consistently choose too low a level of care without being detected. As the number of repetitions of contracting is taken to the limit, the correct punishment strategy can overcome the inefficiencies inherent in asymmetric information.

Radner [1981] used a similar method of attack in a general principal-agent relationship with a large but finite number of repetitions. The principal can observe an outcome that is a function of the agent's action and the state of nature. However, he is unable to separate the two components. Consequently, the principal's pre-announced reward to the agent is based only on the stochastic outcome. The principal must devise some method of detecting cheating that is sufficiently accurate to induce the desired behavior on the part of the agent, yet leads to

false accusations of cheating with very low probability. If the number of repetitions is sufficiently large, Radner proves that both players can come within epsilon of a one-period cooperative solution.

A model of repeated principal-agent relationships in which the reputation of the agent plays a role was developed by Rogerson [1982]. The context of this model is one in which the principal hires an expert agent to do research and development work. Moral hazard problems develop because the agent is both the advisor to the principal with regard to the likelihood of success of a project and the recipient of the contract if the prediction is favorable. As long as the project outcome is in part dependent on some random element, the principal can never determine with certainty whether the agent has been honest. Therefore the agent can maximize the length of his employment by always reporting a high probability of success. Once it is understood that the relationship may be ongoing for a number of periods, however, the principal has some power to induce truthful revelation by announcing that the relationship will cease after a prescribed number of wrong forecasts. This reservation number of permissible mistakes, and hence the length of employment, depends on the accuracy of the agent's private information.

A priori restrictions on the principal's strategy space permit closed-form solutions to the problem and analysis of some interesting comparative statics in Rogerson's model. The principal must terminate the relationship at some point in order to assure truthful revelation of the agent's private information. Improvements in the agent's signal

influence the number of mistakes that the principal permits, but the exact effect depends on whether the improvement is in the success signal or the failure signal. If the agent's ability to predict failures increases, he is less likely to falsely recommend projects. Knowing this, the principal can be more lenient in setting the reservation number of mistakes and permit the relationship to last for a longer time on average. The same is not true when the accuracy of the success signal is enhanced. This refinement induces the agent to take chances until the number of permissible mistakes is reduced to a sufficiently threateningly small number. Consequently, in order to maintain truthful revelation, the principal is forced to reduce the reservation number of mistakes. The reduction in chances exactly offsets the effect of the improvement in the success signal so that the expected length of the relationship is unchanged.

Rogerson's model can be reinterpreted in terms of credit markets. Lenders might find it necessary to threaten to withhold funds after a borrower reaches some threshold number of delinquent payments (mistakes). The long-term specifications of the loan contract would depend in part on the borrower's ability to assess the probability of a favorable outcome and his facility in communicating that information to the lender.

The conflict between the agent's incentives and the principal's objective prohibit first-best efficiency results in single-period principal-agent models. When the relationship is expected to be ongoing for an indefinitely long period, these inefficiencies can be overcome.

However, the number of repetitions needed to achieve the first-best outcome in the models discussed above often makes them somewhat unrealistic. Very little is known about the properties of finitely repeated relationships. Yet an analysis of this sort might contribute to the understanding of some types of transactions in contexts such as credit markets.

Frequently both the lender and the borrower in a credit transaction anticipate negotiating multiple loan contracts over a period of time. A detailed credit history is kept for each borrower, and the terms of the later loans depend on the outcome of the earlier contracts. An experience-rating system is used to determine the terms of each loan extended. Borrowers who make timely repayments of earlier loans tend to receive favorable treatment subsequently. On the other hand, borrowers who are delinquent in their repayments or even default can obtain additional funds only after agreeing to higher interest rates and closer supervision by the lender. In some cases the lender might refuse to continue the relationship entirely. The threat of future punishment brings the borrower's incentives into closer alignment with the lender's goals. Insights from the principal-agent models, particularly those which explicitly incorporate repetition, should be useful in analyzing the borrower-lender relationship.

2.3 Credit Market Literature

Research in the area of credit markets has been dominated by attempts to explain the "anomaly" of credit rationing. Beginning with Roosa in 1951 and extending through the 1970's, a long series of

articles were published, each of which reached the same basic conclusion.^{7/} If the lender is prohibited from charging a different interest rate to different borrowers, credit rationing is rational because the risk of any given loan is not independent of the size of that loan. Hence, borrowers requesting a large sum of money will be "rationed" because the market interest rate is insufficient to compensate for the risk borne by the lender.

These credit models rely on size as a crucial distinguishing feature among borrowers. Although size is, no doubt, of great interest to a lender, it is not clear that risk increases monotonically with the size of the loan. An undercapitalized firm can be a very poor credit risk. As long as no single loan is large enough to constitute a significant proportion of the lender's portfolio, the emphasis on size to the exclusion of all other characteristics is unwarranted. Factors such as the firm's credit history, the quality of the borrowing firm's manager, and the investment plan itself should be regarded as equally important in evaluating creditworthiness of potential borrowers.

The model set forth by Jaffee and Russell [1976] was the first attempt to incorporate the problems of asymmetric information into an analysis of credit markets. Each borrower has private knowledge about the probability of defaulting on a loan; the lender, on the other hand, cannot distinguish borrowers until some have, in fact, defaulted. Consequently, the contract offerings can take on one of two forms. Either all borrowers have their loan size restricted, or the market

oscillates between a single contract and a pair of contracts that allows borrowers to self-select on the basis of probability of repayment.

In the Jaffee-Russell model, borrowers are either honest or dishonest. Honest borrowers always repay their loans; dishonest borrowers repay their loans only when it is less costly to do so than it is to default. Since the cost of repayment rises with the dollar amount borrowed, the probability of default rises with the size of the loan. Competitive lenders are assumed to have access to perfect capital markets. Loanable funds are available at a constant rate. Therefore, the supply curve is horizontal until it reaches the point at which it becomes profitable for some borrowers to default. Thereafter, the shape of the supply curve is determined by the distribution of default probabilities. It could be either upward-sloping or backward-sloping.

Assume that the demand for loanable funds is accurately described by the demand curve in Figure 3(a). If the supply curve is backward-bending throughout the entire range in which default can be expected, the only potential equilibrium will occur at the point where the supply curve starts to rise. All borrowers are rationed at that level since demand exceeds supply at the interest rate $r = i$. Suppose instead that the market has settled at point A in Figure 3(b). A new creditor could enter the market, offering a contract at point B. It can be shown that all honest borrowers prefer the new contract; all dishonest ones prefer the contract at point A. However, the old contract is no longer profitable to the lender when only dishonest borrowers select it because the probability of default is so high. Eventually, this

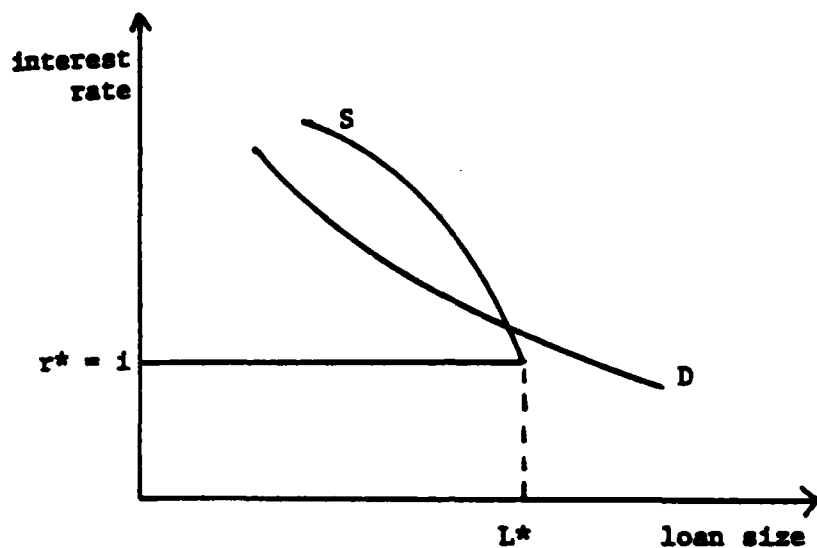


Figure 3(a): Equilibrium in the credit market may involve rationing for all borrowers.

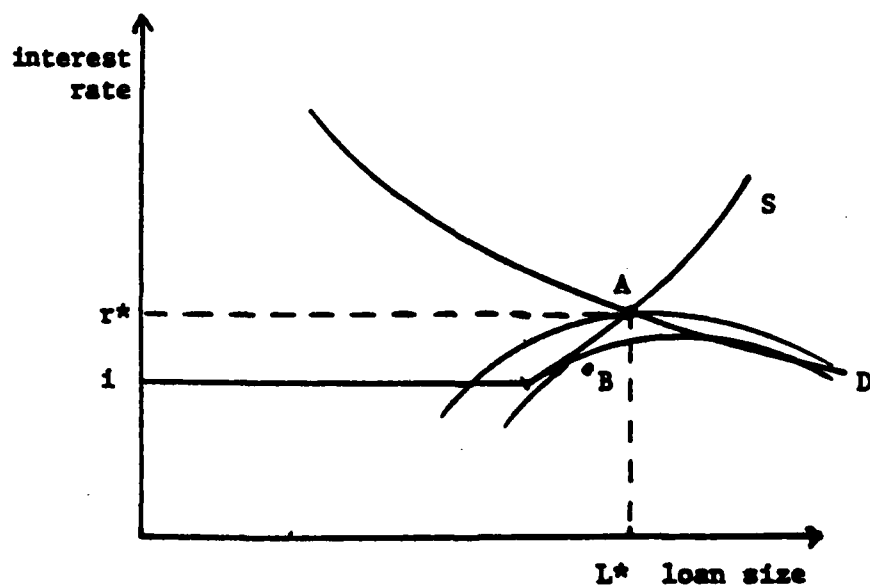


Figure 3(b): Equilibrium in the credit market may not exist.

contract will be dropped from the market, and the contract at point B will have to be modified until it is again on the lender's supply curve.

The Jaffee-Russell model is completely analogous to the Rothschild-Sitglitz insurance model. Equilibrium in the insurance market, when it exists, is characterized by a set of policies that induce self-selection among the different risk classes of consumers. Equilibrium in the credit market cannot exist because, by assumption, the lender is precluded from offering different contracts to different classes of borrowers. Consequently, there is always another loan contract that would be profitable if only the honest borrowers choose it; but once this contract becomes available, the loans extended to dishonest borrowers return expected losses. Just as in the Wilson and Riley models, a greater degree of rationality must be attributed to the lender before a stable equilibrium set of loan contracts can be achieved.

Stiglitz and Weiss [1980] developed a model in which they established circumstances under which credit rationing is a likely response to asymmetric information problems. In their model, all projects are fixed at the same size but differ in the degree of risk involved. Borrowers appear identical a priori, so the lender is unable to discriminate among them. Supply of loanable funds depends on the rate of return paid to depositors. This rate, in turn, is determined by the zero-profit condition for competitive lenders.

The key to Stiglitz and Weiss' analysis is the disparity between the objectives of the lender and the borrowers. The lender's expected profit function is concave in the project realization. The lender not

only bears all of the downside risk, but has a ceiling on the return from any given loan. The borrower's expected return from an investment project is convex in the outcome. The borrower receives nothing if the return is too low to repay the loan; if the return is sufficiently high, the borrower receives everything in excess of the fixed debt owed the lender. All else equal, an increase in risk benefits the borrower but decreases the expected return to the lender.

Even though excess demand for loanable funds exists in the market, lenders might have no incentive to raise the interest rate to clear the market. This is due to the fact that the interest rate serves as a self-selection mechanism. A high interest rate drives the more conservative borrowers out of the market and leaves only the borrowers with riskier projects. Since the lender must bear the burden of any default, the lender's expected profits could fall because the extra risk more than offsets the higher interest rate. Figure 4 illustrates this argument for the case of two types of borrowers. Any interest rate in excess of r_A drives the more conservative group A from the market. The demand from group B alone might be inadequate to support an interest rate as high as r_B . Therefore, a competitive lender would not find it profitable to charge a rate higher than r_A , even though excess demand persists.

In addition to the self-selection problems, moral hazard offers another explanation for a lender's reluctance to raise the interest rate, even in the face of excess demand. If a borrower were indifferent between two projects at interest rate r_1 in Figure 5, increasing the

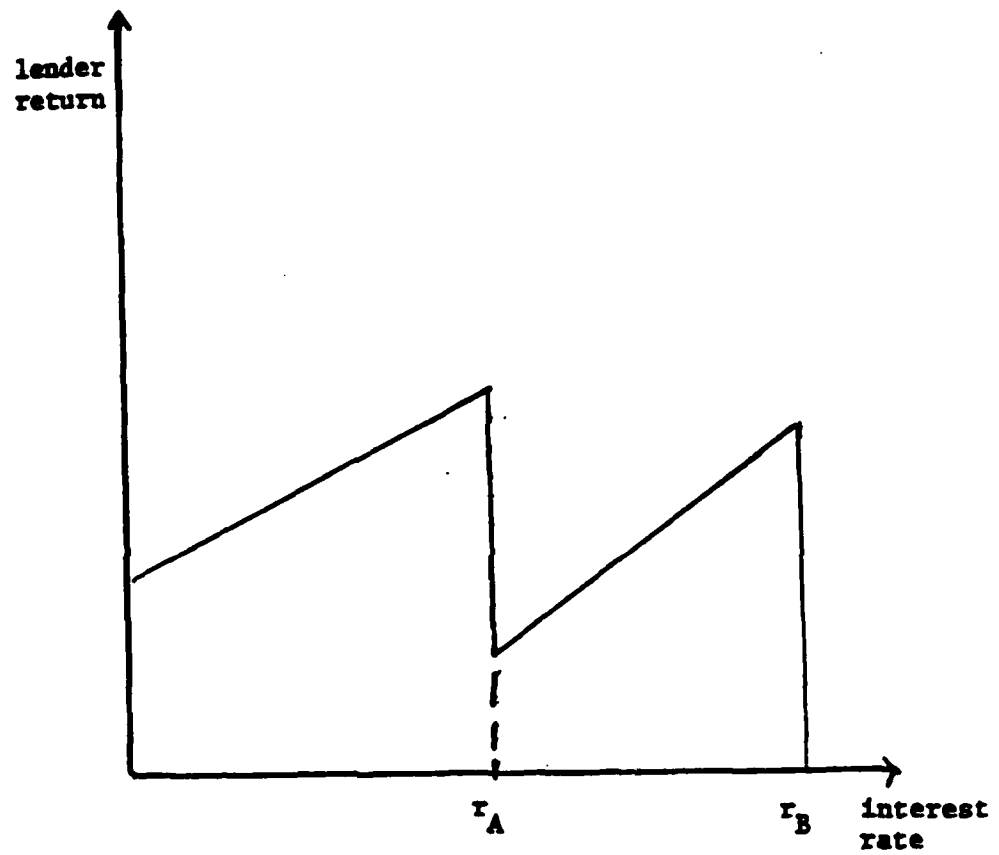


Figure 4: Lenders' returns need not increase monotonically with the interest rate due to adverse selection.

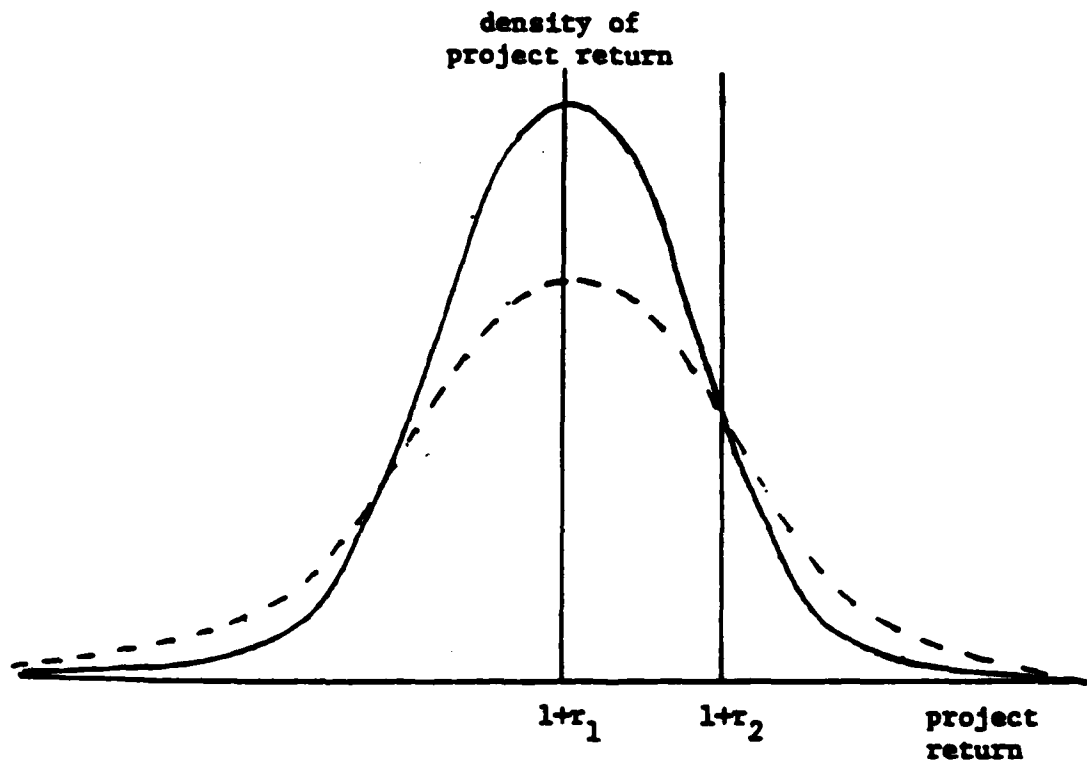


Figure 5: Lenders' returns need not increase monotonically with the interest rate due to moral hazard.

rate to r_2 would cause the same borrower to prefer the more risky project since only the upper tail of the distribution of returns affects the debtor's returns. Once again, the increase in risk could outweigh the lender's gain from the higher interest rate.

Adding collateral as another instrument available to the lender is not sufficient to eliminate the possibility of credit rationing. Collateral forces the borrower to pay for some of the downside risk of an investment project, making default less likely and increasing the return to the lender when default does occur. However, collateral also has a sorting effect. More cautious potential borrowers might choose not to invest rather than shoulder the extra risk of losing the collateral. Again, the lender's loss of the more profitable borrowers could more than counter the gain from the collateral requirement.

The results in the Stiglitz-Weiss model are due to the fact that no mechanism is available that permits the lender to distinguish one potential borrower from another. Just as in the Wilson model, the lender knows only market averages. The lender cannot increase expected profits by raising the interest rate because the lower-risk firms would leave the market. If it were possible to gain some knowledge of the risk involved in each investment project, the lender's dilemma would be alleviated to some extent. It could charge a lower interest rate to investors who can demonstrate creditworthiness and raise the interest rate for the remaining potential borrowers.

A second part of Stiglitz-Weiss [1981] analysis investigates rationing of another sort. When repeated contracting is anticipated,

lenders can influence the behavior of investors by making extensions of future loans contingent on repayment of past debts. As argued above, a low interest rate can induce investment in safer projects. It might also lead to positive expected profits for the borrowing firms. The lender can exploit the desire of the borrowers to retain high profits by threatening to withhold funds if the first loan is not repaid in full. If second-period debt is subsidiary to outstanding first-period debt, the competitive argument that the borrower can always find another lender is inoperative.

Credit market models have added considerable insight into the complexities of loan transactions. However, they are still far from adequate. Recent work such as the Stiglitz-Weiss model has abandoned the attempt by lenders to categorize borrowers into risk classes so that appropriate contract terms can be offered to each group. Very little effort has been made to identify instruments that can effectively sort borrowers. Furthermore, there have been very few attempts to explicitly study the consequences of multiple transactions between the same two agents.

2.4 Dissertation Model^{8/}

The purpose of the thesis is to assess the relative importance of signaling and repeated contracting in the context of borrower-lender relationships. Each of these devices can aid the lender in identifying observationally identical borrowers. Two bodies of theory have analyzed each of these implements in isolation. The thesis investigates the implications and consequences of using both signaling and repetition

simultaneously. If both parties expect the duration of the relationship to be infinite, with probability one the lender will eventually learn the borrowers' private information; there would be no justification for using resources to devise signaling mechanisms. In this model, the number of repetitions is assumed to be finite a priori. Therefore, both identification mechanisms will be used in equilibrium.

Virtually all of the existing literature pertaining to credit markets assumes a competitive market structure. A new borrower might view potential creditors as extremely competitive; yet, once a relationship has been developed between that borrower and a particular lender, the borrower could find itself essentially tied to that institution.

In terms of efficiency, a trade-off between market power and repeated contracting must be faced. Some degree of monopoly power is necessary in order to enforce the mechanisms devised to counteract the self-serving incentives of the riskier borrowers. Monopoly power has a socially beneficial role since the threats of one lender are not credible if loanable funds are readily available from a second source. As a matter of simplification, therefore, the lender has been modeled as a monopolist in the same sense that the principal in a principal-agent model wields market power. Consequently, the problems with breakdown of potential equilibria prevalent in competitive signaling models do not arise in the dissertation model. The lender is the sole source of credit in the market, so no new contracts can be proposed to upset the initial set of equilibrium contracts.

The ability of the lender to gain some information about the technology employed by each potential borrower is an essential aspect of the model developed in the dissertation. The importance of size of the loan is tempered; instead, the analysis concentrates on the effectiveness of other signaling mechanisms in mitigating the lender's informational disadvantage. The signal is modeled rather abstractly as a factor share that critically impacts productive efficiency. It can be interpreted as a measure of the business acumen of the entrepreneur, the division of the entrepreneur's time between business duties and engineering activities, or the degree of care expended in running the business. All of the power to identify technological type is collapsed into the factor share parameter in the one-period model; when repetition is introduced, the first-period project outcome can be used in conjunction with the factor share to aid the lender in determining profit-maximizing contracts for each borrower.

The dissertation model is similar to the Spence labor market model in that only two types of technology are introduced. Hence, the average riskiness of investment projects plays a much less important role than in the Akerlof and Wilson signaling models. The lender can always devise contracts that distinguish the types, so it is not forced to rely on averages. However, identification is costly. The lender will choose to offer only an "average" contract whenever the cost of signaling is great enough to outweigh the gains. Both pooling and separating contracts will be examined in an attempt to establish the conditions that determine the lender's optimal contract offerings.

The primary feature that distinguishes the dissertation model from the rest of the signaling literature is the analysis of the effectiveness of the signal in repeated contracting. The signal serves two roles in determining the lender's expected profit over the two periods. It directly affects the profit from the loans extended in the initial period. It also is instrumental in determining the quality of information available to the lender at the start of the second period since the distribution of the size of the project outcomes depends on the factor shares. The lender faces a non-trivial trade-off between expected profit in the first period from relatively efficient use of technology and increases in expected profit in the final period because of more accurate information from the initial project returns.

The fact that the lender gains some knowledge about technological type simply by observing the project outcomes in the first period complicates the determination of the profit-maximizing contracts. The lender must take account of the interactions between the two signals, the contract choice and the first-period outcome, when constructing its loan offers. Early acquisition of information is beneficial. However, the high-risk firms require more compensation for revealing their identities at the outset of repeated transactions since they lose their informational advantage for a longer period of time. If restrictions are placed on the form of threats and promises available to the lender, it could prove more profitable to wait until the second period before attempting to unambiguously sort the borrowers on the basis of technological type.

The conclusions that follow from analysis of a one-period model do not necessarily hold when repetition is allowed. The complexities of the trade-offs from one period to the next can make some decisions optimal in a repeated relationship even though they would be suboptimal in a single transaction. Since many business relationships are expected to endure multiple but finite transactions, investigation of comparable economic models is warranted.

3. Introduction to the Variable-Loan Size Model

The market studied in this essay is characterized by a single lender and a known number of firms of each of two different technological types. Each entrepreneur has complete knowledge of the technological abilities of his firm; the lender, on the other hand, cannot costlessly classify any given borrower. Since these two technology types differ in the riskiness of their investment projects, the lender would like to offer different contract terms to each type. However, this separation can be accomplished only by sacrificing some productive efficiency. An effort will be made to establish circumstances under which the benefits from identifying each firm's technological type outweigh the costs.

The basic model of one credit transaction between the lender and each borrower is developed in Section 4. The contracts that the lender would offer if it had access to the same information as the firms are derived as a base case. These are referred to as the complete information contracts. In an environment of asymmetric information, the lender

can either offer two contracts, each of which will be selected by only one type of firm (separate), or offer one contract to all borrowers (pool). It will be shown that a pooling contract is always dominated by a pair of separating contracts in the one-loan model. Nevertheless, the pooling optimum will be characterized in the one-loan model because the results will be useful in the more complicated multiple-transaction model.

The separating contracts offered for a single transaction have the properties of most signaling models. The contracts must be devised in such a way that the high-risk firms will be satisfied to accept the contracts intended for them. To accomplish this, the high-risk firms are offered more attractive terms than in the complete information case. The incentive problem imposes an externality on the low-risk firms. They are forced to pay for this externality in two ways. The interest rate charged them is higher than it would be with complete information. Furthermore, they must overinvest in one input in order to signal their identities, thus leading to inefficient production.

When the lender decides to pool all loan applicants, the determination of the actual contract becomes more complicated. The pooling interest rate always lies between the two complete information interest rates, regardless of the form of the contract. The same is not true for the factor share. If the lender's expected return from both technology types is positive under the terms of the optimal pooling contract, the input share is a sort of weighted average of the productively efficient coefficients for each type. However, if the lender's expected return

from the high-risk firms is sufficiently low, the lender might choose to require a factor share that forces all borrowers to overinvest in one of the inputs.

Sections 5 and 6 investigate the implications of signaling combined with repeated transactions. The planning horizon is expanded to only two transactions, but this is sufficient to capture the essence of the gains from repetition. The properties of the equilibrium in two different cases will be studied. In the first case, the lender is restricted to commitments that will be profit-maximizing at every stage of negotiation. In the second case, no restrictions are imposed on the nature of the commitment.

If the commitments permitted the lender are restricted, the rewards and punishments of the separating contracts become even more extreme than in the single-transaction case. In effect, the lender must offer terms at the beginning of the first loan that induce the cooperation of the firms for two stages of contracting instead of just one. The restrictive nature of the commitments can make repeated transactions disadvantageous for the lender.

Repeated transactions can be beneficial when the lender is free to make any form of commitment to its borrowers. The lender has more flexibility in formulating the separating contracts within the context of a standing relationship because the terms of the second contract can be made contingent upon the outcome of the first loan. These terms can incorporate extreme threats to firms that are found

guilty of misrepresenting themselves. Such threats are credible when no alternative sources of funding are available.

4. One-Loan Model with Loan Size Variable

Throughout this section of the paper, only single transactions between the lender and any given firm are considered. The issues of standing relationships and commitments do not arise in this context; however, an understanding of the one-loan model greatly simplifies the analysis of the model in which repeated transactions are allowed. The standing relationship between the lender and borrower will be explicitly incorporated into the two-loan model in Sections 5 and 6.

The lender in this model specializes in offering production loans to small business firms. The firms in the market are of one of two types, α or β , where each type is distinguished by the technology employed in its production processes. In particular, the optimal mix of inputs differs between firms with different technologies. The objective of every agent in the model is to maximize its own expected profit.

The probability of default on any given loan is always positive. In return for assuming this default risk, the lender is rewarded with a risk premium, r . (This will be referred to as an interest rate even though the concept of time is not relevant in the one-loan model.) A different degree of risk is consistently associated with each of the two technologies used by the firms. Therefore, ideally, the lender would like to be able to identify each firm's technology a priori and adjust the interest rate accordingly. The problem facing the lender is that of

optimally distributing funds among the firms and setting the interest rates it charges in such a way that the lender's total expected return is maximized.

Specifically, the distribution of returns for any given type of firm is common knowledge for the lender and all of the firms in the market even before any loans are extended. It is also known to all that there are a total of N_α of the type α firms and N_β of the type β firms in the market. However, the true identity of any particular borrower is known only to that firm. One or more types of firms might find it advantageous to conceal their true characters from the lender; if any one type prefers anonymity, all firms will be precluded from credibly revealing their identities.

Ordinarily, one would expect that both the borrower and the lender would benefit from having the firm determine the division of funds among its variable inputs since the firm has the advantage of knowing its true type. In this model, however, the intrinsic relationship between the firm's optimal input mix and its associated degree of risk allows the lender to infer information about the firm's true technology type from its input choice. The division of funds to which the firm will agree not only determines the efficiency of the production, but also conveys useful information about the firm's type to the uninformed lender. Therefore, the lender retains the right to offer loan contracts which stipulate not only the interest rate charged, but also the requisite division of funds among the inputs. The firm will be allowed to choose

among these contracts and to determine the size of the loan, given its choice of contracts.

An incentive problem arises once the firm is aware that the lender is using its production plan as a signal of its true identity. The firm's input selection might be altered away from its first-best efficient point in order to qualify for a loan contract that permits a lower interest rate. In this sense, the signal is only an imperfect indication of the firm's true type. In the one-loan model, the input mix is the only signal available to the lender. In the two-loan model, the lender will be able to gain further knowledge about the firm by establishing a standing relationship with it and observing the return to its first loan.

It is implicitly assumed that the size of the firm is not perfectly correlated with its type in order to prevent the size of the loan demanded from functioning as an additional signal of the firm's technology. The model attempts to capture some of the salient features of financing the operations of small firms in their start-up stages. Therefore, the lender's a priori notion of sound business management has greater import than its perception of the optimal scale. In principle, a firm making use of a particular technology could be either small or large and still be a good credit risk. Since the intent of this paper is to focus on firm attributes such as good management rather than size, per se, the variability in scale of firm will not be explicitly incorporated in the text. Instead, the scale factor which enters the production can be thought of as varying among firms of any given technology

type; the scale factor that appears in the lender's expected profit function can be thought of as a market average for each type of technology. Of course, if the size of the loan demanded did transmit information about the firm's technology, the lender might choose to institute some form of credit rationing. Therefore, by excluding this possibility, this model implicitly assumes that the size of a firm's operation conveys little or no pertinent information to the lender.

The production technology of a firm of type α is given by the Cobb-Douglas function^{9/}

$$(4.1) \quad \omega_{\alpha 1} = \theta_{\alpha} \gamma_{\alpha} x_{\alpha 1}^{\alpha} y_{\alpha 1}^{\frac{1}{2} - \alpha},$$

where

- $\omega_{\alpha 1}$ = output of a type α firm from loan 1;
- $x_{\alpha 1}$ = expenditure on variable input x from loan 1;
- $y_{\alpha 1}$ = expenditure on variable input y from loan 1;
- γ_{α} = an exogenously given fixed factor of production associated with technology α ;
- α = the exogenously given exponent of the x input for firm type α ; $\alpha \in (0, \frac{1}{2})$;
- θ_{α} = a random variable with a continuous distribution function $F_{\alpha}(\theta_{\alpha})$; $F_{\alpha}(0) = 0$.

Note that the firm's type, α , is also the exponent on the x input and, therefore, serves as a measure of the "x-intensity" of this particular technology.

The fact that the sum of the exponents of $x_{\alpha 1}$ and $y_{\alpha 1}$ is restricted to be less than one assures that the production technology displays decreasing returns to scale. This assumption, together with the positivity restriction on α , guarantees that the firm's optimal choice of both variable inputs is neither zero nor infinite in the short run.

Type β firms follow the Cobb-Douglas technology

$$(4.2) \quad \omega_{\beta 1} = \theta_{\beta} \gamma_{\beta} x_{\beta 1}^{\frac{1}{2} - \beta} y_{\beta 1}^{\beta},$$

where all of the variables are defined analogously to those in the type α production function. It will be assumed that $\beta < \alpha$. In other words, the technology characterizing type α firms requires a greater expenditure on the x input than that used by type β firms. This assumption will be maintained without any loss of generality because symmetric arguments apply for the case $\beta > \alpha$ throughout the analysis.

All firms in this model are entirely dependent on debt financing to cover the costs of their variable productive inputs. The dollar amount of a loan extended to a type α firm is divided between its expenditures on variable inputs x and y in the fractions

$$(4.3) \quad x_{\alpha 1} = s_{\alpha 1} l_{\alpha 1}$$

and

$$(4.4) \quad y_{\alpha 1} = (1 - s_{\alpha 1}) l_{\alpha 1},$$

where $s_{\alpha 1} \in (0,1)$ and $l_{\alpha 1}$ is the dollar amount of the loan granted to the firm. Using these equations, a type α firm's production function can now be written in terms of the size of the loan as follows:

$$(4.5) \quad \omega_{\alpha 1} = \theta_{\alpha} \gamma_{\alpha} s_{\alpha 1}^{\alpha} (1 - s_{\alpha 1})^{\frac{1}{2} - \alpha} l_{\alpha 1}^{\frac{1}{2}} .$$

In so specifying the division of funds, the price of both inputs has been implicitly set at one dollar. For simplicity, the price of the output is also normalized to one. The interest rate remains the only endogenous price in the model.

Although the assumption of all-debt financing of the firm's investment projects might seem unrealistic, extending the model by allowing alternative means of financing would only serve to shift the issues at hand into another securities market. For example, if equity financing replaced the loans in the model, the stockholders would want access to accurate information about the firms in order to assess the optimal allocation of their funds. The same sort of incentive problems would arise in the stock market as in the loan market modeled in this paper. In addition, it is assumed that no internal financing is available for any of the firms. This is reasonable in the context of startup firms; in such circumstances, the project financed by the loan is essentially equivalent to the firm. These simplifying assumptions are made in order to highlight the key issues of interest. Alternatives to strictly debt financing would complicate, but not resolve, the incentive problems.

Each firm of type j is assumed to receive all of the revenue from its investment project above the level $(1 + r_{j1})l_{j1}$, the cost of its obligation to the lender. If the return to the project is insufficient to cover this amount, it receives nothing. The firm never suffers negative profits since no collateral requirement has been built into this model. Therefore, the firm's realized profit is given by:

$$(4.6) \quad \pi_{j1} = \max\{0, \omega_{j1} - (1 + r_{j1})l_{j1}\}, \quad j = \alpha, \beta$$

In exchange for lending money to type j firms at the rate of $(1 + r_{j1})$ and assuming the risk of default, the lender receives the entire return to the firm's project up to a limit of $(1 + r_{j1})l_{j1}$. The lender pays a market-determined rate of $(1 + i)$ for the use of these funds, where $i < r_{j1}$. Funds from this market are not available to an individual firm because its risk of default is too great; the lender, on the other hand, is able to minimize this risk by dealing with many small firms whose random project returns are uncorrelated. This yields the lender's realized return from a single type j firm:

$$(4.7) \quad \rho_{j1} = \min\{\omega_{j1}, (1 + r_{j1})l_{j1}\} - (1 + i)l_{j1}, \quad j = \alpha, \beta$$

4.1 Complete Information Case

If the lender were able to freely and accurately classify each loan applicant before granting any funds, the choice of the optimal contract parameters would be straightforward. The lender would calculate its expected profits using the same distribution of returns as the

firm employs. It is helpful to examine the outcome of the loan contracts offered in an environment of complete information as a basis of comparison to the results derived in the asymmetric information case. The complete information calculations for a type β firm run exactly parallel to those for a type α firm. Therefore, the analysis will be conducted only with respect to a typical type α firm.

Since the lender in this model is the sole source of funds for the firms, it acts as a monopolist in setting the terms of the loan contracts $(s_{\alpha 1}, r_{\alpha 1})$. Given the type, the lender can use the firm's expected profit function to derive its demand function for loanable funds in terms of the contract parameters. Then, knowing the reaction of the firm to any combination of contract terms, the lender can set these parameters to maximize its own total expected return. We will assume that if the lender can positively identify a firm, it can set contract parameters such that its expected return from that firm's loan is non-negative.^{10/} Otherwise, the lender would refuse to extend a loan to such a firm regardless of the contract terms.

A typical type α firm chooses the size of its loan to maximize its expected profits:

$$\begin{aligned}
 (4.8) \quad E\pi_{\alpha 1} &= \max_{l_{\alpha 1}} \int_{\theta_{\alpha}^*}^{\infty} [\theta_{\alpha} \gamma_{\alpha} s_{\alpha 1}^{\alpha} (1 - s_{\alpha 1})^{\frac{1}{2} - \alpha} l_{\alpha 1}^{\frac{1}{2}} - (1 + r_{\alpha 1}) l_{\alpha 1}] f_{\alpha}(\theta_{\alpha}) d\theta_{\alpha} \\
 &= \max_{l_{\alpha 1}} [\gamma_{\alpha} s_{\alpha 1}^{\alpha} (1 - s_{\alpha 1})^{\frac{1}{2} - \alpha} l_{\alpha 1}^{\frac{1}{2}} E(\theta_{\alpha} | \theta_{\alpha} > \theta_{\alpha}^*) - (1 + r_{\alpha 1}) l_{\alpha 1}] [1 - F_{\alpha}(\theta_{\alpha}^*)]
 \end{aligned}$$

where θ^* is defined by the equation

$$(4.9) \quad \theta_{\alpha}^* \gamma_{\alpha} s_{\alpha 1}^{\alpha} (1 - s_{\alpha 1})^{\frac{1}{2} - \alpha} l_{\alpha 1}^{\frac{1}{2}} = (1 + r_{\alpha 1}) l_{\alpha 1} .$$

In other words, θ_{α}^* is the value of the random variable above which the firm begins to earn positive realized profit. The conditions $F(\theta_{\alpha}^*) > 0$ and $[1 - F_{\alpha}(\theta_{\alpha}^*)] > 0$ will be imposed on the project return for the sake of insuring the existence of an economic problem. The first condition states that there is a positive probability of non-repayment of the loan; the second guarantees a positive probability of non-negative realized profit for the firm.

As will be shown shortly, the cut-off value of the random component, θ_{α}^* , does not depend on the loan size chosen by the firm. Hence, the firm's profit function is concave in the loan size over the range $\theta_{\alpha} > \theta_{\alpha}^*$. Concavity, together with the first-order condition, is sufficient to determine the expected profit maximum. The firm's demand function for loanable funds in terms of the input coefficient, $s_{\alpha 1}$, and the interest rate charged for borrowed funds, $r_{\alpha 1}$, is obtained by differentiating equation (4.8) with respect to $l_{\alpha 1}$:

$$(4.10) \quad l_{\alpha 1} = \frac{\gamma^2 [E(\theta_{\alpha} | \theta_{\alpha} > \theta_{\alpha}^*)]^2 s_{\alpha 1}^{2\alpha} (1 - s_{\alpha 1})^{1-2\alpha}}{4(1 + r_{\alpha 1})^2}$$

$$= \frac{A s_{\alpha 1}^{2\alpha} (1 - s_{\alpha 1})^{1-2\alpha}}{4(1 + r_{\alpha 1})^2} ,$$

where A_α is defined as

$$(4.11) \quad A_\alpha = \gamma_\alpha^2 [E(\theta_\alpha | \theta_\alpha > \theta_\alpha^*)]^2 .$$

Equation (4.11) can be interpreted as a measure of the individual type α firm's unalterable marginal efficiency. It is composed of a term that is specific to the particular firm and one that is common to all firms with technology α .

As long as the firm is allowed to remain on its demand curve, the cut-off value of θ_α is independent of the loan contract parameters. Substituting the loan demand function into equation (4.9), we are left with

$$(4.12) \quad \theta_\alpha^* = \frac{1}{2} E(\theta_\alpha | \theta_\alpha > \theta_\alpha^*) .$$

θ_α^* depends only on the exogenous parameters of the probability distribution. Therefore, the break-even value of θ_α is stationary for all firms with technology α , regardless of the contract parameters set by the lender. This simple formula results from the multiplicative form of the random component in the firm's production function.

The firm's expected profits in terms of the loan contract parameters, $(s_{\alpha 1}, r_{\alpha 1})$, can finally be computed by substitution of equation (4.10) into equation (4.8):

$$(4.13) \quad E\pi_{\alpha 1} = \frac{A_\alpha s_{\alpha 1}^{2\alpha} (1 - s_{\alpha 1})^{1-2\alpha} [1 - F_\alpha(\theta_\alpha^*)]}{4(1 + r_{\alpha 1})} .$$

Note that the firm's expected profit is monotonically decreasing in $r_{\alpha 1}$ and is concave in $s_{\alpha 1}$, the maximizing value of which is $s_{\alpha 1} = 2\alpha_1$. If a type α firm were offered a loan contract with no stipulation other than the interest rate, it would always choose to allocate the fraction 2α of its loan money to the x input.

The lender next substitutes the firm's demand function, equation (4.10), into its own expected return function and maximizes over both the interest rate and the factor share coefficient:

$$\begin{aligned}
 (4.14) \quad E\rho_{\alpha 1} &= \max_{(s_{\alpha 1}, r_{\alpha 1})} \int_0^{\theta_{\alpha}^*} \theta_{\alpha} \gamma_{\alpha} s_{\alpha 1}^{\alpha} (1 - s_{\alpha 1})^{\frac{1}{2} - \alpha} \frac{1}{2} f_{\alpha}(\theta_{\alpha}) d\theta_{\alpha} \\
 &\quad + \int_{\theta_{\alpha}^*}^{\infty} (1 + r_{\alpha 1}) \frac{1}{2} f_{\alpha}(\theta_{\alpha}) d\theta_{\alpha} - (1 + i) \frac{1}{2} \\
 &= \max_{(s_{\alpha 1}, r_{\alpha 1})} \frac{A_{\alpha} s_{\alpha 1}^{2\alpha} (1 - s_{\alpha 1})^{1-2\alpha}}{2(1 + r_{\alpha 1})} \\
 &\quad \times \left[\frac{F_{\alpha}(\theta_{\alpha}^*) E(\theta_{\alpha} | \theta_{\alpha} \leq \theta_{\alpha}^*) + \frac{1}{2} [1 - F_{\alpha}(\theta_{\alpha}^*)] E(\theta_{\alpha} | \theta_{\alpha} > \theta_{\alpha}^*)}{E(\theta_{\alpha} | \theta_{\alpha} > \theta_{\alpha}^*)} - \frac{1}{2} \frac{1 + i}{1 + r_{\alpha 1}} \right] \\
 &= \max_{(s_{\alpha 1}, r_{\alpha 1})} \frac{A_{\alpha} s_{\alpha 1}^{2\alpha} (1 - s_{\alpha 1})^{1-2\alpha}}{2(1 + r_{\alpha 1})} [R_{\alpha} - \frac{1}{2} C(r_{\alpha 1})] ,
 \end{aligned}$$

where R_{α} and $C(r_{\alpha 1})$ are defined as

$$(4.15) \quad R_{\alpha} = \frac{F_{\alpha}(\theta_{\alpha}^*) E(\theta_{\alpha} | \theta_{\alpha} \leq \theta_{\alpha}^*) + \frac{1}{2} [1 - F_{\alpha}(\theta_{\alpha}^*)] E(\theta_{\alpha} | \theta_{\alpha} > \theta_{\alpha}^*)}{E(\theta_{\alpha} | \theta_{\alpha} > \theta_{\alpha}^*)}$$

and

$$(4.16) \quad C(r_{\alpha 1}) = \frac{1+i}{1+r_{\alpha 1}} .$$

Equation (4.15) defines the lender's expected return per unit of the firm's efficiency. It depends solely on the parameters of the distribution of θ_{α} since θ_{α}^* is independent of the contract parameters. The term $C(r_{\alpha 1})$ can be interpreted as the cost of funds to the lender per dollar of return from the borrower. It is the reciprocal of the lender's markup for borrowed funds. It is important to note that $C(r_{\alpha 1})$ is a decreasing function of the interest rate charged by the lender.

The contract parameters derived from the lender's complete information expected profit function imply that the lender maximizes its expected return by allowing the firms to minimize their production costs and by equating its own marginal cost with expected marginal revenue. After simplifying the first derivatives of equation (4.14) with respect to the factor share coefficient and the interest rate, the optimal complete information contract parameters, $(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1})$, are determined as

$$(4.17) \quad \bar{s}_{\alpha 1} = 2\alpha$$

and

$$(4.18) \quad C(\bar{r}_{\alpha 1}) = R_{\alpha} .$$

Equation (4.17) demonstrates that each firm is allowed to maximize its productive efficiency. As noted before, 2α is also the optimal value of the factor share from the firm's point of view. The lender's and the

firm's optimal productive input coefficient coincide in a complete information context because this is simply a first-best productive efficiency condition. Equation (4.18) determines the optimal markup charged by the monopolist lender, taking account of the borrower's reactions to the contract parameters. Note that the complete information interest rate rises as the cost of funds to the lender, $(1 + i)$, rises and as the probability of default, $F_{\alpha}(\theta_{\alpha}^*)$, increases. The lender demands compensation commensurate with the expense and the risk of any given loan.

4.2 Asymmetric Information Case

The above analysis has demonstrated that in a market comprised of more than one type of firm, the lender would like to discriminate among the firms and offer different contracts of the form (s, r) to each individual type. However, to do this optimally, the lender would need to be able to categorize the firms accurately before extending any loan money. One or more firm types might find it in their own interests to falsify their true identity in order to qualify for more lenient loan contract terms. That is, a firm might be willing to distort its factor input ratio away from its productive optimum in order to obtain a lower interest rate on its loan.

Hereinafter we will maintain the assumption that firms of type β would prefer to be granted the complete information loan contracts intended for the type α firms, $(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1})$, to their own complete information contracts, $(\bar{s}_{\beta 1}, \bar{r}_{\beta 1})$. Recall that the break-even value of θ_{β} is independent of the loan contract parameters; therefore, θ_{β}^* is the

same regardless of which contract the type β firms choose. Denoting a type β firm's loan demand and expected profit with a type α contract as $l_{\beta}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1})$ and $E\pi_{\beta}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1})$, respectively, and with its own contract terms as $l_{\beta}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1})$ and $E\pi_{\beta}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1})$, this condition will hold if the exogenous parameters are fixed such that

$$\begin{aligned}
 (4.19) \quad E\pi_{\beta}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1}) &= \int_{\theta_{\beta}^*}^{\infty} \{ \theta_{\beta} \gamma_{\beta} \bar{s}_{\alpha 1}^{\beta} (1 - \bar{s}_{\alpha 1})^{\frac{1}{2} - \beta} [l_{\beta}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1})]^{\frac{1}{2}} \\
 &\quad - (1 + \bar{r}_{\alpha 1}) l_{\beta}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1}) \} f_{\beta}(\theta_{\beta}) d\theta_{\beta} \\
 &= \frac{A_{\beta} (2\alpha)^{2\beta} (1 - 2\alpha)^{1-2\beta}}{4(1 + \bar{r}_{\alpha 1})} [1 - F_{\beta}(\theta_{\beta}^*)] \\
 &> \int_{\theta_{\beta}^*}^{\infty} \{ \theta_{\beta} \gamma_{\beta} \bar{s}_{\beta 1}^{\beta} (1 - \bar{s}_{\beta 1})^{\frac{1}{2} - \beta} [l_{\beta}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1})]^{\frac{1}{2}} \\
 &\quad - (1 + \bar{r}_{\beta 1}) l_{\beta}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1}) \} f_{\beta}(\theta_{\beta}) d\theta_{\beta} \\
 &= \frac{A_{\beta} (2\beta)^{2\beta} (1 - 2\beta)^{1-2\beta}}{4(1 + \bar{r}_{\beta 1})} [1 - F_{\beta}(\theta_{\beta}^*)] \\
 &= E\pi_{\beta}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1}) ,
 \end{aligned}$$

which reduces to the condition

$$(4.20) \quad \frac{(2\alpha)^{2\beta} (1 - 2\alpha)^{1-2\beta}}{(2\beta)^{2\beta} (1 - 2\beta)^{1-2\beta}} > \frac{1 + \bar{r}_{\alpha 1}}{1 + \bar{r}_{\beta 1}} .$$

Intuitively, condition (4.20) states that a firm with type β technology would prefer the complete information contract intended for a type α firm if the relative efficiency with type α factor shares is greater than the relative interest rates charged to firms characterized by the two types of technologies. The type β firm would save more on the interest rate than it would lose due to inefficient production with the type α contract.

We have assumed the existence of an incentive problem; namely, that type β firms prefer the complete information contract of type α firms to their own. In fact, this incentive problem can operate in only one direction. That is, given that $E\pi_{\beta}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1}) > E\pi_{\beta}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1})$, it cannot be true that a type α firm would prefer a type β contract to its own in a complete information context. To demonstrate this point, first suppose that, to the contrary, $E\pi_{\alpha}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1}) > E\pi_{\alpha}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1})$. This, in turn implies that

$$(4.21) \quad \frac{(2\alpha)^{2\alpha}(1-2\alpha)^{1-2\alpha}}{(2\beta)^{2\alpha}(1-2\beta)^{1-2\alpha}} < \frac{1 + \bar{r}_{\alpha 1}}{1 + \bar{r}_{\beta 1}}$$

This condition can be interpreted analogously to inequality (4.20). To see that the lender faces no incentive problem with the type α firms, note that the LHS of inequality (4.20) is less than one while the LHS of inequality (4.21) exceeds one. Conditions (4.20) and (4.21) cannot hold simultaneously. The reasoning behind this conclusion is clear. It has already been determined that $\bar{s}_{\beta 1}$ was set at its productively efficient point, so it must be true that $E\pi_{\beta}(\bar{s}_{\beta 1}, \cdot) \geq (E\pi_{\beta}(\bar{s}_{\alpha 1}, \cdot))$. Therefore, under the maintained assumption, it must follow that

$\bar{r}_{\alpha 1} < \bar{r}_{\beta 1}$, and that the inequality is great enough to more than compensate for the loss in profits arising from the distortions of factor inputs away from their efficient point. Similarly, we know that $E\pi_{\alpha}(s_{\alpha 1}, \cdot) \geq E\pi_{\alpha}(\bar{s}_{\beta 1}, \cdot)$ because $\bar{s}_{\alpha 1} = 2\alpha$ is the productively efficient point for type α firms. The profit functions for firms of both technology types are monotonically decreasing in the interest rate, so $E\pi_{\alpha}(\cdot, \bar{r}_{\alpha 1}) \geq E\pi_{\alpha}(\cdot, \bar{r}_{\beta 1})$. Since type α firms prefer both terms of their own complete information contract to the terms of a type β contract, we have $E\pi_{\alpha}(\bar{s}_{\alpha 1}, \bar{r}_{\alpha 1}) \geq E\pi_{\alpha}(\bar{s}_{\beta 1}, \bar{r}_{\beta 1})$. The property we have just established can be summarized as follows:

In the one-loan model, the incentive problem in the market caused by dishonest firms can run in only one direction. That is, if firms with type β technology prefer the complete information contracts intended for firms characterized by type α technology to their own, then it cannot be true that type α firms prefer the type β loan contracts.

In light of the above result, the lender will be concerned with the externalities arising only from the firms with type β technology in modeling the particular problems that arise below.

Faced with the inability to distinguish firm technologies and knowing that both type α and type β firms would prefer to be granted the complete information contract terms intended for type α firms, the lender must choose between two possible strategies. It could offer two different sets of contracts but distort the terms of these contracts away from the ones that would obtain with complete information in such a way that the type β firms would no longer find the type α contract terms more profitable. This option would allow the lender to continue

to separate the two groups of firms but at the expense of either devising type β contracts that are more attractive or type α contracts that are less attractive to the type β firm than the corresponding complete information contracts would be. Alternatively, the lender could pool all of the loan applicants and offer only one set of contract terms to any firm seeking funds. One might expect the terms of such a contract to be more or less a weighted average of the terms of the two sets of complete information contracts. This strategy would eliminate the incentive problem of impostor type β firms posing as type α firms since no attempt would be made to distinguish the two groups.

The remaining portion of this section will examine the optimal contracts under first the separating and then the pooling strategies. In a one-loan model, it will be demonstrated that a rational lender always offers separating contracts to its borrowers. This conclusion, however, is not obvious until the properties of both strategies have been determined. Moreover, a characterization of the optimal pooling contract is worthwhile because it will simplify the analysis of the more complicated model developed in subsequent sections.

4.3 Separating Contracts

Analysis of the separating strategy involves an investigation of the loan contracts that obtain when the lender is unable to identify the technology of any given firm but still finds it profitable to devise contracts such that each type of firm will choose the one designed for it. That is, the lender must set the contract parameters to meet the constraint

$$(4.22) \quad E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) \geq E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) ,$$

where the carets are used to indicate the optimal contract terms under asymmetric information conditions. Once again, the lender can calculate the size of the loan demanded by each type of firm in terms of the contract parameters. Then, summing over the expected returns from the loans when each firm selects its own separating contract and weighting by the number of each type of firm, the lender can choose contract parameters $(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1})$ and $(\hat{s}_{\beta 1}, \hat{r}_{\beta 1})$ to maximize the function

$$(4.23) \quad E\rho = \max \text{ wrt } (s_{\alpha 1}, r_{\alpha 1}, s_{\beta 1}, r_{\beta 1}, \lambda)$$

$$\begin{aligned} & \sum_{j=\alpha, \beta} N_j E\rho_j(s_{j1}, r_{j1}) + \lambda N_{\beta} [E\pi_{\beta}(s_{\beta 1}, r_{\beta 1}) - E\pi_{\beta}(s_{\alpha 1}, r_{\alpha 1})] \\ &= \max \sum_{j=\alpha, \beta} \frac{N_j A_j s_{j1}^{2j} (1 - s_{j1})^{1-2j}}{2(1 + r_{j1})} [R_j - \frac{1}{2} C(r_{j1})] \\ & \quad + \frac{\lambda}{4} N_{\beta} A_{\beta} [1 - F_{\beta}(\theta_{\beta}^*)] \left[\frac{s_{\beta 1}^{2\beta} (1 - s_{\beta 1})^{1-2\beta}}{1 + r_{\beta 1}} - \frac{s_{\alpha 1}^{2\beta} (1 - s_{\alpha 1})^{1-2\beta}}{1 + r_{\alpha 1}} \right] , \end{aligned}$$

where λ is the Lagrange multiplier associated with the constraint and the expectation operator is used with respect to θ_{α} and θ_{β} .

The change in each firm type's expected profits under the separating scheme relative to the complete information case can be ascertained by an examination of the first-order conditions of equation (4.23). The honest type α firms are not only charged a higher interest rate under

the separating contracts, but they are also forced to operate with inefficient factor shares in order to satisfy the required signal of their true technology type. Thus, the type α firms are driven to lower expected profit levels because of the presence of the potentially dishonest firms in the market. In contrast, the type β firms are charged a lower interest rate; furthermore, they are allowed to minimize the costs of production by operating efficiently. The parameters for the separating loan contracts compensate the firms with type β technology for revealing their identities to the lender. Results of this sort are standard in signaling models.

It is easy to establish the properties of the interest rates charged under the lender's optimal separating contracts. Differentiating the lender's constrained profit function with respect to the interest rates for the type α and the type β firms, respectively, we obtain

$$(4.24) \quad [R_{\alpha} - C(\hat{r}_{\alpha 1})] = \frac{\lambda}{2} \frac{N_{\beta}^A \hat{s}_{\alpha 1}^{2\beta} (1 - \hat{s}_{\alpha 1})^{1-2\beta}}{N_{\alpha}^A \hat{s}_{\alpha 1}^{2\alpha} (1 - \hat{s}_{\alpha 1})^{1-2\alpha}} [1 - F_{\beta}(\theta_{\beta}^*)]$$

and

$$(4.25) \quad [R_{\beta} - C(\hat{r}_{\beta 1})] = -\frac{\lambda}{2} [1 - F_{\beta}(\theta_{\beta}^*)] .$$

Equation (4.24) implies that $\hat{r}_{\alpha 1}$ is greater than $\bar{r}_{\alpha 1}$. The RHS of the equality is positive. This is obvious on inspection for all terms except λ , the Lagrange multiplier. However, we have assumed that the constraint is always binding; therefore, λ is always greater than

zero. In the complete information case, the expression $[R_\alpha - C(r_{\alpha 1})]$ was equated to zero. It is set equal to a positive number when the lender chooses to separate. Recalling that $C(r_{\alpha 1})$ is a decreasing function of the interest rate, it follows that $\hat{r}_{\alpha 1} > \bar{r}_{\alpha 1}$. In contrast, equation (4.25) demonstrates that the difference between the marginal revenue and marginal cost for firms with type β technology is now negative. Reversing the previous argument, we obtain the result that $\hat{r}_{\beta 1} < \bar{r}_{\beta 1}$. That is, the lender charges a lower interest rate to the type β firms when it is unable to distinguish firms' technologies than it would with complete information.

The lender's purpose in devising separating loan contracts is to charge each firm an interest rate more nearly commensurate with the degree of risk associated with its project than is possible with a pooling contract. Therefore, the lender's control of an instrument in addition to the interest rate is imperative in achieving separation of the firm types. In this model, the factor input coefficient serves that role. Knowing that firms with type α technology use relatively more of the x input than do firms of type β , the lender will require a firm to agree to a high value of s before qualifying for a low interest rate. In this way, the lender can learn the true identity of each of its loan applicants by its choice of contracts.

In order to discriminate between the two technology types in the market, the lender must force the relatively x -intensive type firms to overinvest in the x input. Once again, this claim can be supported by

examining the first-order conditions of the lender's expected profit function. Differentiating equation (4.23) with respect to $s_{\alpha 1}$ yields:

$$(4.26) \quad [R_{\alpha} - \frac{1}{2} C(\hat{r}_{\alpha 1})] \\ = \frac{\lambda}{2} \frac{N_{\beta} A_{\beta} \hat{s}_{\alpha 1}^{2\beta} (1 - \hat{s}_{\alpha 1})^{1-2\beta}}{N_{\alpha} A_{\alpha} \hat{s}_{\alpha 1}^{2\alpha} (1 - \hat{s}_{\alpha 1})^{1-2\alpha}} [1 - F_{\beta}(\theta_{\beta}^*)] \left[\frac{2\beta - \hat{s}_{\alpha 1}}{2\alpha - \hat{s}_{\alpha 1}} \right].$$

Subtracting equation (4.24) from equation (4.26) we obtain:

$$(4.27) \quad C(\hat{r}_{\alpha 1}) = \lambda \frac{N_{\beta} A_{\beta} \hat{s}_{\alpha 1}^{2\beta} (1 - \hat{s}_{\alpha 1})^{1-2\beta}}{N_{\alpha} A_{\alpha} \hat{s}_{\alpha 1}^{2\alpha} (1 - \hat{s}_{\alpha 1})^{1-2\alpha}} [1 - F_{\beta}(\theta_{\beta}^*)] \left[\frac{2\beta - 2\alpha}{2\alpha - \hat{s}_{\alpha 1}} \right].$$

From equation (4.27) it follows that the optimal factor share, $\hat{s}_{\alpha 1}$, exceeds 2α . The LHS of this equation is positive, as is the entire expression on the RHS preceding the term

$$(4.28) \quad \left[\frac{2\beta - 2\alpha}{2\alpha - \hat{s}_{\alpha 1}} \right].$$

Therefore, the assumption that $\beta < \alpha$ implies that $2\alpha < \hat{s}_{\alpha 1}$. In other words, the lender's optimal separating contract forces the honest firms to signal with a factor input choice more heavily concentrated toward the x input than is required for efficient production. This is the analog of the conclusion drawn from the typical signaling model. The agent relatively more efficient in the factor used as a signal is forced to overinvest in that factor.

The firms with type β technology will be allowed to select an efficient input mix when the lender follows a separating policy. Under the assumption that type β firms prefer the complete information contract of the type α firms, we already know that type α firms must prefer their own complete information contract. Since there is no need for the type β separating contract to play any deterrence role, the optimal choice of contract parameters will specify efficient division of funds between the variable inputs for firms with type β technology. That is, the lender sets $\hat{s}_{\beta 1} = 2\beta$. This follows directly upon differentiating equation (4.23) with respect to $s_{\beta 1}$.

Despite the increase in the interest rate for the type α firms and the decrease in the interest rate for the type β firms relative to the complete information contracts, $\hat{r}_{\alpha 1}$ is still less than $\hat{r}_{\beta 1}$. This can be determined readily by simplifying the constraint:

$$(4.29) \quad \frac{\hat{s}_{\beta 1}^{2\beta} (1 - \hat{s}_{\beta 1})^{1-2\beta}}{(1 + r_{\beta 1})} = \frac{\hat{s}_{\alpha 1}^{2\beta} (1 - \hat{s}_{\alpha 1})^{1-2\beta}}{(1 + r_{\alpha 1})} .$$

We have just shown that the numerator on the LHS of equation (4.29) attains its maximum value. Therefore, we must have $\hat{r}_{\alpha 1} < \hat{r}_{\beta 1}$ in order to satisfy the equality.

Before offering the separating loan contracts to the firms, the lender must ascertain that a new incentive problem has not arisen; namely, that the distortions inherent in the separating contracts have not left the type β loan more profitable than the type α terms for firms with type α technology. The inequality

$$(4.30) \quad \frac{\hat{s}_{\alpha 1}^{2\alpha} (1 - \hat{s}_{\alpha 1})^{1-2\alpha}}{1 + \hat{r}_{\alpha 1}} \geq \frac{\hat{s}_{\beta 1}^{2\alpha} (1 - \hat{s}_{\beta 1})^{1-2\alpha}}{1 + \hat{r}_{\beta 1}}$$

must be preserved under the separating contract parameters. Multiplying each side of inequality (4.30) by the corresponding side of equation (4.29) and rearranging terms produces the result

$$(4.31) \quad \left| \frac{\hat{s}_{\alpha 1}}{\hat{s}_{\beta 1}} \right|^{2(\alpha-\beta)} \geq \left| \frac{1 - \hat{s}_{\alpha 1}}{1 - \hat{s}_{\beta 1}} \right|^{2(\alpha-\beta)},$$

which is always true since $\hat{s}_{\alpha 1} > 2\beta = \hat{s}_{\beta 1}$. Hence, the contracts determined by the maximization of equation (4.23) are in fact separating contracts.

The optimal separating contracts in a one-loan model require the lender to coax the dishonest type β firms into compliance with a reduced interest rate while encumbering the honest type α firms with less desirable contract terms than would be optimal with complete information. The type α firm is forced to accept a contract which comprises not only a higher interest rate than $\bar{r}_{\alpha 1}$, but also a distortion of its productive inputs, resulting in lower expected returns for both the type α firms and the lender. The lender's expected returns are greater with the complete information contracts than with the separating loan contracts. However, the inability to distinguish a firm's technology a priori makes these contracts infeasible.

In summary, the important properties of the lender's optimal one-period separating contracts can be stated as follows:

The type α firms must pay a higher interest rate relative to the complete information contract and overinvestment in the x input when the lender chooses a separating strategy in a one-loan model. In addition to maintaining efficient production, the type β firms are rewarded with a lower interest rate, leaving them with strictly greater expected profits than in the complete information case.

As stated earlier, a pair of separating contracts can be found to dominate every pooling contract when only one loan transaction is anticipated. Once the properties of the pooling contract have been ascertained, this claim will be substantiated.

4.4 Pooling Contracts

When the lender chooses the pooling strategy, all firms applying for loans are offered the same contract terms, (s_{pl}, r_{pl}) . The lender knows the size of the loans that will be demanded by each type of firm in terms of the pooling contract parameters. It also knows the total number of firms characterized by each of the two types of technologies. Therefore, the lender can calculate its total expected profit by summing over its expected return from each particular type of firm, weighted by the number of such firms in the market. The lender then selects the single pair of contract terms $(\hat{s}_{pl}, \hat{r}_{pl})$ that maximizes its total expected return:

$$\begin{aligned}
 (4.32) \quad E_p &= \max_{(s_{pl}, r_{pl})} \sum_{j=\alpha, \beta} N_j E_{pj}(s_{pl}, r_{pl}) \\
 &= \max_{(s_{pl}, r_{pl})} \sum_{j=\alpha, \beta} \frac{N_j A_j S_{pl}^{2j} (1 - s_{pl})^{1-2j}}{2(1 + r_{pl})} \left[R_j - \frac{1}{2} C(r_{pl}) \right].
 \end{aligned}$$

The first-order conditions for this maximization problem yield interesting properties of the optimal pooling interest rate and factor input coefficient. In particular, the pooling interest rate, r_{pl} , will always assume some intermediate value between the complete information interest rates of the two types of technologies. Simplifying the derivative of the lender's expected profit function with respect to the interest rate, we obtain

$$(4.33) \quad C(\hat{r}_{pl}) = \sum_{j=\alpha,\beta} \frac{w_j}{w_\alpha + w_\beta} R_j ,$$

where the weights are defined as

$$(4.34) \quad w_j = N_j A_j \hat{s}_{pl}^{2j} (1 - \hat{s}_{pl})^{1-2j} , \quad j = \alpha, \beta .$$

As one might expect, this condition verifies that the pooling interest rate must lie between the type α and the type β complete information interest rates. The relative cost of funds, $C(\hat{r}_{pl})$, is a weighted average of the corresponding complete information costs for each of the types of technologies. This follows directly from a comparison of equations (4.18) and (4.33). Two components determine the weights for each firm type. The first component is a measure of the technological efficiency when operating with the factor share \hat{s}_{pl} ; the second is the total number of firms with technology j in the market.

To see that the optimal pooling interest rate takes on an intermediate value between $\bar{r}_{\alpha 1}$ and $\bar{r}_{\beta 1}$, recall that the maintained assumption that each type β firm prefers the complete information contract intended for the type α technology to its own complete information

contract led to the conclusion that $\bar{r}_{\alpha 1} < \bar{r}_{\beta 1}$. From this and equation (4.33) it follows that

$$(4.35) \quad C(\bar{r}_{\alpha 1}) > C(\bar{r}_{pl}) > C(\bar{r}_{\beta 1}) .$$

Noting that $(1 + r_{j1}) = \frac{1+i}{C(r_{j1})}$, we can conclude that

$$(4.36) \quad \bar{r}_{\alpha 1} < \bar{r}_{pl} < \bar{r}_{\beta 1} .$$

One might further anticipate that the optimal input coefficient \hat{s}_{pl} in the pooling contract would also take on an intermediate value between the complete information input coefficients of the two types of firms. The conditions under which this is true can be ascertained from an examination of the first-order condition of the lender's profit function with respect to s_{pl} :

$$(4.37) \quad \frac{2\alpha - \hat{s}_{pl}}{2\beta - \hat{s}_{pl}} = \frac{-N A \hat{s}_{pl}^{2\beta} (1 - \hat{s}_{pl})^{1-2\beta} [R_{\beta} - \frac{1}{2} C(\hat{r}_{pl})]}{N A \hat{s}_{pl}^{2\alpha} (1 - \hat{s}_{pl})^{1-2\alpha} [R_{\alpha} - \frac{1}{2} C(\hat{r}_{pl})]} .$$

As long as the lender is able to extract positive expected returns from the loans to both types of firms, the bracketed expressions in both the numerator and denominator of the RHS of equation (4.37) are positive; therefore, the entire RHS is negative. Hence, we can establish that the lender's optimal pooling factor input ratio lies between the complete information ratios of the respective firms when the loans to both types of firms generate positive expected returns for the lender.

The pooling input coefficient will be forced outside the interval between the complete information coefficients for the two technology

types if the lender faces expected losses from the type β firms under the pooling contract. The relative cost of funds term is a decreasing function of the interest rate, so $\hat{r}_{\alpha 1} < \hat{r}_{\beta 1}$ assures that the expected returns from the type α firms are always positive when the lender chooses to pool. However, once the pooling contract is established, it is possible that the lender could face expected losses from the type β firms since $\hat{r}_{\beta 1} < \hat{r}_{\alpha 1}$. Through an analysis that is just the reverse of the argument in the previous paragraph, we can establish that in this case, the optimal pooling factor share lies outside the interval $[2\beta, 2\alpha]$. Furthermore, a continuity argument supports the claim that the optimal pooling input coefficient now exceeds 2α . First, assume that the lender is earning positive expected profits from both firms (the RHS of equation (4.37) is negative). Holding the type α production parameters constant, move the parameters of the type β firms continuously such that the quantity on the LHS of equation (4.37) moves from positive to negative values. As long as this quantity is positive, we have $2\beta < \hat{s}_{\beta 1} < 2\alpha$. When the expected return from a loan to a type β firm is exactly zero, $\hat{s}_{\beta 1} = 2\alpha$. Throughout the exercise, $\beta < \alpha$. Therefore, when the expected return from a loan to a type β firm becomes negative, we have $2\beta < 2\alpha < \hat{s}_{\beta 1}$.

The power of the pooling contract in effecting the lender's goals derives from the concavity of the firms' profit functions with respect to the input coefficient. The lender is able to cut its losses from the type β firms by driving them down their demand curves without unduly disrupting the productivity of the type α firms by setting $\hat{s}_{\beta 1}$

slightly higher than 2α but relatively far from 2β . This feat cannot be accomplished by altering the interest rate, however, since both firm types' expected profits are monotonically decreasing in the interest rate. Cutting expected losses from type β firms by charging a high interest rate would simultaneously reduce expected profits from the type α firms by approximately the same degree.

The result that the x factor-intensive firm might be forced to invest in more than the optimal amount of input x is similar to Spence's conclusion about overinvestment in education.^{11/} However, in the preceding case the overinvestment is not induced by any incentive constraint since the incentive problem was eliminated with the decision to pool all loan applicants. Rather, this overinvestment in the x factor arises as the least costly method of cutting the lender's expected losses from the firms it would like to exclude if it were able to identify them.

We have established the following properties of the lender's optimal one-period pooling contract:

The optimal interest rate in the pooling contract always lies between the complete information interest rates for each of the firm types. The optimal factor share lies between the complete information shares when the lender's expected return from firms with both types of technologies is positive. However, if the expected returns from type β firms are negative under the pooling contract, the optimal input coefficient exceeds both 2β and 2α . Even though the lender has chosen a pooling strategy, both types of firms are forced to overinvest in the input used as a signal.

It is now simple to demonstrate graphically that at least some pair of separating contracts always provides a higher expected return for the lender than a single pooling contract in the one-loan model. Suppose that point A in the top plate of Figure 6 is determined as the optimal pooling contract. The interest rate always lies between the complete information interest rates for the respective technologies. At point A, the input coefficient also lies in the interval $[2\beta, 2\alpha]$. The lender could increase its expected return without reducing the expected profit for any firm by adding a contract at point B to its list of offerings. The type β firms would be indifferent between the contract at point A and the new one at point B. The type α firms would prefer the original contract at a point A. Obviously, the lender's expected return from the type α firms is unchanged, but the expected return from the type β firms under the terms of the contract at point B is increased. The improvement in the productive efficiency for the type β firms allows the lender to raise the type β interest rate without reducing the expected profit for any firm. Note that the contracts at points A and B separate the technology types. However, they need not be the optimal separating contracts. In other words, there could exist another pair of separating contracts that yield even greater additions to the lender's expected returns.

A similar argument holds when the optimal pooling contract dictates overinvestment for all firms. Point C in the lower plate of Figure 6 exemplifies such a contract. Again, the lender could offer a

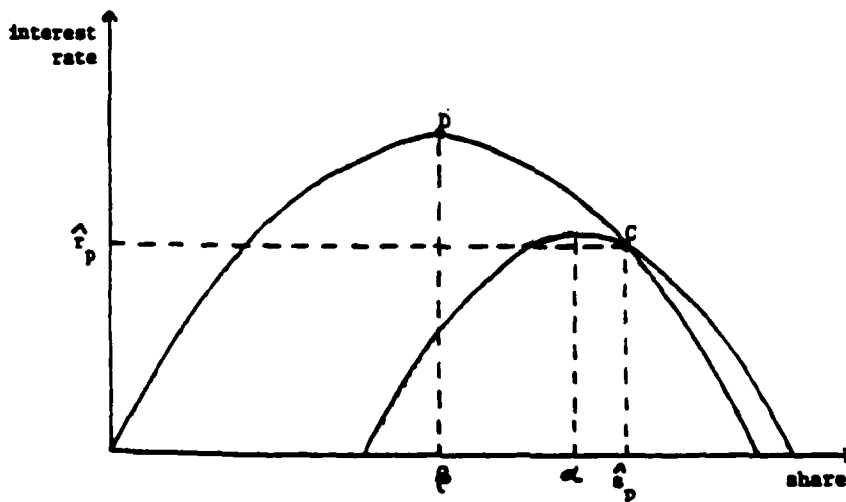
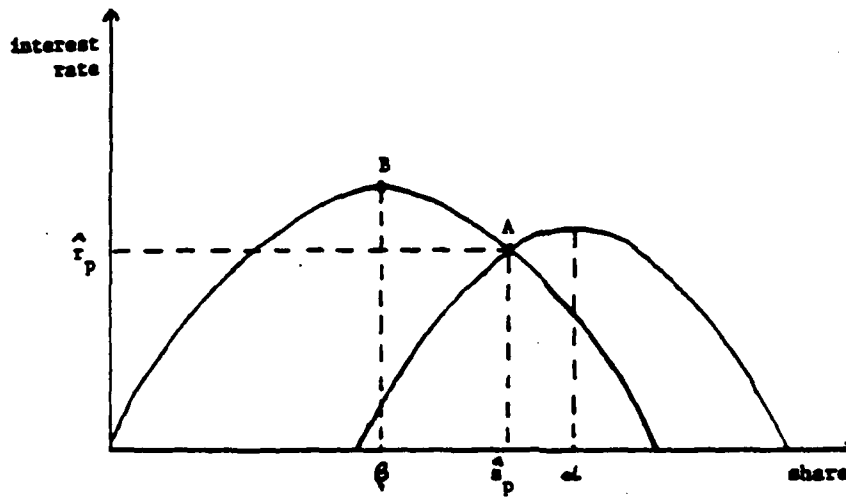


Figure 6: Pooling contracts are dominated by separating contracts for a single transaction in the variable-loan-size model.

new contract on the same isoprofit curve for type β firms at point D. No borrower would lose in terms of expected profit under this scheme, but the lender would achieve higher total expected returns because of the increased efficiency in production for type β firms.

5. Perfect Strategies in a Two-Loan Model with Loan Size Variable

In the two-loan model, the lender is still allowed to select both the interest rate and the input coefficient for every loan it extends. The lender's available information before granting the first loan is the same as it was in the one-loan model. In addition, it obtains an accurate report of the realized outcome of the project financed by each of the first loans before it is obligated to offer terms for the second loan to any of the borrowers. This means that the lender is at liberty to further segment its loan applicants by returns to their first project and make the terms of the second loan dependent on the first project outcome. However, because the project realizations are random, the information gained by the lender is not necessarily perfect information. The project outcome alone might not reveal the true technology of any given firm.

A major dichotomy in the lender's possible strategies concerns the issue of whether it can make threats and promises as part of the terms of early loans that influence the terms of later loans offered to the same borrower. For example, the lender might threaten to withhold funds for a second loan altogether if a particular firm tried to misrepresent its type when it applied for the first loan and the project outcome

revealed the firm's dishonesty. Alternatively, the lender might agree to relatively lenient contract terms for one or more loans in an effort to procure a firm's cooperation in sharing information about its technology. The lender's ability to commit itself to either threats or promises significantly improves its ability to elicit private information from the firms, thereby enabling the lender to earn higher expected returns even from the loans granted in the first round.

Both perfect and imperfect lender strategies will be investigated. Perfect strategies, discussed in this section, restrict the lender to self-enforcing threats and promises about the second loan contracts. Self-enforcing commitments are ones which would be optimal for the lender to implement on the basis of the expected return from the second loan only, where the expectation incorporates the information gained from the first project outcome. While the information learned by the lender from the first project return can never be harmful, it will be shown that under the stringent restriction of perfect strategies, standing relationships between the lender and borrowers might actually diminish the expected returns of the lender. Separation at the beginning of the two-loan horizon could be so costly that a first-period pooling contract becomes viable.

Imperfect strategies, those in which unrestricted threats and promises can be used by the lender, will be studied in Section 6. Here the lender can commit itself to contract terms even if there is some chance that these terms will prove to be less than profit maximizing once the project return from the first loan is known. This option

clearly cannot decrease the value of a standing relationship for the lender because it simply increases the set of feasible contract choices. Perfect strategies are still permitted; the option of ignoring the consequences of the repetition is also open. In fact, with imperfect strategies the lender might be able to strictly improve its position by the establishment of long-term relationships with borrowing firms.

Both the separating and the pooling contracts will be examined when the lender is restricted to perfect strategies at the outset of a two-period horizon. However, it will be argued that only separating contracts are relevant when imperfection is permitted. As long as any form of commitment is credible, the lender can offer whatever set of second-period contracts would optimally follow a first-period pooling contract. Just as in a one-loan context, however, some productive inefficiencies can be eliminated by offering separating rather than pooling contracts in the first period.

Attention will be focused on the possible gain the lender can derive from the maintenance of standing relationships with its clients. It should be understood that the improvement under study is that which might arise from the long-term relationship in addition to the increase in information from observation of the outcome of the initial project. The fact that the lender benefits from knowledge of the credit history of its borrowers is easily understood. The purpose here is to establish whether this is the sole source of advantage resulting from the standing relationship or whether other components contribute to its value.

Throughout the remainder of Sections 5 and 6, the following notation will be used:

ω = a particular project outcome feasible for one or both types of technology. $\omega = k\theta$ where

$$k = \frac{\gamma_j^2 s_j^{2j} (1 - s_j)^{1-2j} E(\theta_j | \theta_j > \theta_j^*)}{2(1 + r_j)} .$$

$g_\alpha(\omega)$ = the density function of the first-stage project outcomes for firms with type α technology.

$g_\beta(\omega)$ = the density function of the first-stage project outcomes for firms with type β technology.

Ω = the support for first-stage project realizations which are feasible for one or both types of technologies ($g_\alpha(\omega) > 0$ or $g_\beta(\omega) > 0$).

Ω' = the subset of the project support defined by outcomes which are feasible for both types of technologies ($g_\alpha(\omega) > 0$ and $g_\beta(\omega) > 0$). $\Omega' \subset \Omega$.

The upper and lower bounds of this subset are defined by $\bar{\omega}$ and $\underline{\omega}$, respectively (if $\bar{\omega}$ exists).

p_α = the probability that a firm with type α technology will be unambiguously identified by the return to its first project. $p_\alpha = \int_{\underline{\omega}}^{\infty} g_\alpha(\omega) d\omega$.

p_β = the probability that a firm with type β technology will be unambiguously identified by the return to its first project. $p_\beta = \int_0^{\underline{\omega}} g_\beta(\omega) d\omega$.

No discounting is incorporated into the model since it only serves to complicate the algebra without adding insight into the analysis.

When restricted to perfect strategies, the lender can make only self-enforcing threats and promises with regard to the contract terms of the second loan. For example, any firm whose first project realization unambiguously reveals its identity must be offered a complete information loan contract in the second stage, regardless of any threats or promises that might have been made. Since pooling contracts are dominated in a one-loan model, all unidentified firms in the second period will be offered separating contracts. However, the restrictions on the lender's decisions can make separating at the outset such a costly endeavor that a pooling contract in the first period might lead to the maximum two-period expected return for the lender.

In this section, we will examine the optimal contracts that obtain when the lender chooses the separating strategy as well as when it chooses the pooling strategy at the beginning of the two-loan horizon. In addition, the outcome of each strategy will be compared to that which

would result from a model in which no standing relationships were allowed, but the outcome of each firm's first project could be costlessly learned by the lender before granting terms for a second loan. The purpose of this comparison is to determine whether any benefit accrues to the lender from the maintenance of standing relationships with its clients over and above the improvement in information. In the comparison model, the lender can make no commitments, either harmful or beneficial, to the borrowers. In effect, it treats each loan as an isolated transaction. However, in determining the terms of the second loan, the lender can calculate its expectations conditionally on the outcome of the first-period project.

5.1 Separating Contracts for the First Loan

We will investigate the potential gain derived from standing relationships if the lender offers separating contracts to the firms that apply for the first loan.^{12/} It will be shown that a perfect strategy that calls for separation in the first stage can be ruled out as a mechanism through which the lender finds a long-term relationship valuable beyond the acquisition of technological information.

When the lender chooses to separate the firms in the first stage of a two-loan model, perfectness imposes very stringent structure on the problem. Given that the lender has successfully separated the firms with the contracts offered for the first loans, it has positively identified each firm's type by the end of the first stage. The lender's optimal decision when the technology is known unambiguously is to offer the complete information contracts. Therefore, the perfectness

requirement of always maximizing the remaining expected return, given the available information, completely determines the outcome of the second loan.

The lender's choice of the first-loan separating contract parameters must assure that each type β firm's expected profit over two stages of accepting the contracts designed for its technology is greater than or equal to its expected profit from accepting the contracts offered to a type α firm at each of the two stages. That is, the lender must meet the constraint

$$(5.1) \quad E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) + E\pi_{\beta}(\bar{s}_{\beta 2}, \bar{r}_{\beta 2}) \\ \geq E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + E\pi_{\beta}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2}) .$$

Throughout this discussion we have maintained the assumption that type β firms find the complete information contracts intended for firms with type α technology more profitable than their own. Therefore, the parameters pertaining to the first loans must be set such that $E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1})$ is strictly greater than $E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1})$. From this we can anticipate that the optimal lender's strategy in a two-loan problem is less likely to call for separating contracts for the first loan than would be the case in the analogous one-loan problem.

To determine the lender's optimal contract parameters, the expected returns from each firm characterized by a particular type of technology at each stage are weighted by the known number of such firms in the market and summed. No probability densities for the first project returns need to be evaluated since there is no uncertainty about the

firms' identities after the first loans have been accepted. The constrained maximand takes the form

$$(5.2) \quad E\rho = \max_{j=\alpha,\beta} \left\{ N_j [E\rho_j(s_{j1}, r_{j1}) + E\rho_j(\bar{s}_{j2}, \bar{r}_{j2})] \right. \\ \left. + \lambda N_\beta [E\pi_\beta(s_{\beta 1}, r_{\beta 1}) + E\pi_\beta(\bar{s}_{\beta 1}, \bar{r}_{\beta 1}) - E\pi_\beta(s_{\alpha 1}, r_{\alpha 1}) - E\pi_\beta(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2})] \right\} .$$

The first-order conditions with respect to the contract parameters appear functionally exactly as they did in the one-loan problem and therefore have been omitted. The derivative with respect to the Lagrange multiplier, λ , is the only functional difference between the system of equations derived from the two-loan problem and the system generated by the one-loan problem.

Consider just the returns from the first loan of the two-loan problem. If the lender finds it optimal to separate here, it will also want to separate in the one-loan model. We have already noted that $E\pi_\beta(\bar{s}_{\beta 2}, \bar{r}_{\beta 2}) - E\pi_\beta(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2})$ enters the constraint as a negative constant. As the constraint becomes more difficult to satisfy, the Lagrange multiplier increases in magnitude. Since the functional form of the lender's expected returns has been altered only by the addition of a constant, the fact that the Lagrange multiplier has become larger is sufficient to conclude that the lender's expected profits from the first loans extended have decreased.

The distortions in the contract parameters away from the complete information contracts are exacerbated as the type α contract becomes more attractive to the firms with type β technology. Such a change makes the incentive constraint more difficult to satisfy, and therefore

even more dramatic inducements must be offered to achieve separation. These changes in the separating contract parameters can be studied by first noting that the Lagrange multiplier increases as the constraint becomes more tightly binding. To see that both the interest rate and the factor share for the type α firms are increased as λ increases, one can define $\hat{s}_{\alpha 1}$ and $\hat{r}_{\alpha 1}$ as implicit functions of λ and totally differentiate equations (4.24) and (4.26) from the one-loan problem. The effect of an increase in λ on the type β contract parameters is easier to determine. As was argued in the one-loan separating model, the input coefficient for the type β firms is independent of λ . These firms continue to operate efficiently. In addition, their interest rate decreases as the constraint becomes more difficult to satisfy. This follows easily from equation (4.25).

In those cases in which the lender is restricted to perfect strategies and chooses to separate at the outset of a two-loan problem, the pertinent features of the optimal loan contracts can be summarized as follows:

In order to achieve separation in the first stage of a two-loan problem in which only perfect strategies are feasible, the distortions in the contract parameters operate in the same direction but are more extreme than those of the one-loan problem. The firms characterized by type α technology must suffer even greater losses in the first period because of the externalities inflicted by the dishonest firms. The type β firms receive inflated payoffs in the form of lower interest rates in addition to being allowed to produce efficiently. The second loan contract parameters are determined entirely by the perfectness requirement. All firms receive their complete information contracts for their second loans.

In the above case, the lender derives no benefit from standing relationships with the borrowers apart from the improvement in information about the firms' technologies. Its expected return is no higher than it would be from two one-loan contracts in which the lender was endowed with the same information sets to which it has access in the two-loan problem. That is, before granting the first loan, the lender would not be able to identify the technology used by any firm; in the second one-loan problem, all of the firms would be accurately classified. The argument above leads to the conclusion that the return from the first loan is decreased in the two-loan model while the return from the second loan is identical under either structure. Hence, under the constraint of perfect strategies, the lender's informational gain from long-term relationships with borrowers is offset by the high level of incentive payoffs necessary to induce separation. In fact, it is possible for the cost to be great enough to outweigh the gain from more accurate classification of technological types.^{13/}

5.2 Pooling Contracts for the First Loan

A pooling strategy is optimal for the lender when there is no net gain to be made from any sort of commitments. All firms are treated identically insofar as the first loan is concerned. After learning the project outcomes financed by the first loans, the lender groups the firms by the size of their project returns and then optimally sets the contract terms for the second loan within each such group.

Dynamic programming can be implemented to solve this two-loan problem. Starting with the second loan, the lender uses all the

information it has learned from the first loan to maximize its remaining expected return. The identity of some firms will be completely determined with positive probability as long as the supports of the random project returns of the two technology types do not overlap entirely. If a firm's type is revealed after the first stage, the lender maximizes its expected return from that firm by granting a second loan based on the complete information parameters. With probability p_α any given type α firm will be perfectly identified by its first project outcome. The lender can therefore expect to award $p_\alpha N_\alpha$ complete information type α contracts in the second round. Similarly, a single type β firm will be completely revealed by its first outcome with probability p_β ; so on average $p_\beta N_\beta$ of the second loans will be complete information type β contracts. These contracts, of course, are exactly the same in the two-loan problem as they were in the one-loan problem. If the borrower loses its informational advantage, it has no choice but to cooperate with the lender. The lender has no need to distort the contract parameters away from the first-best solution.

The remaining $(1 - p_\alpha)N_\alpha + (1 - p_\beta)N_\beta$ firms are expected to realize project outcomes that could be generated by either the type α or the type β technology. Having observed both the size of the first loan requested and the first-period project outcome, the lender has sufficient information to calculate the likelihood of each type of firm's achieving any particular outcome w in this range. Then, for each of these possible outcomes, a problem essentially the same as a one-loan problem must be solved. The only difference here is

that the expected return from each technology type is weighted by the corresponding density function, $g_j(\omega)$, evaluated at the particular outcome ω .

In Section 4, it was demonstrated that pooling contracts are dominated when only one loan contract is relevant. Consequently, only separating contracts will be offered to firms with first-stage project outcomes in the region Ω' . The equations pertaining to the separating contracts differ from the one-period separating problem only in that the respective probability densities multiply the total number of firms characterized by each of the two technology types. That is, the lender's problem in the second period for firms with a particular outcome can be stated as:

$$(5.3) \quad E_p^2 = \max_{j=\alpha, \beta} \sum_j g_j(\omega) N_j E_p(s_{j2}(\omega), r_{j2}(\omega)) \\ + \lambda(\omega) g_\beta(\omega) N_\beta [E\pi_\beta(s_{\beta 2}(\omega), r_{\beta 2}(\omega)) - E\pi_\beta(s_{\alpha 2}(\omega), r_{\alpha 2}(\omega))] .$$

The first-order conditions for this equation appear in Appendix A. As is readily seen, the distortions in the separating contract terms depend on the likelihood ratio

$$(5.4) \quad \frac{g_\beta(\omega)}{g_\alpha(\omega)} = \frac{g_\beta | \theta_\beta \frac{\gamma_\beta^2 s_{\beta 1}^{2\beta} (1 - s_{\beta 1})^{1-2\beta} E(\theta_\beta | \theta_\beta > \theta_\beta^*)}{2(1 + r_{\beta 1})}}{g_\alpha | \theta_\alpha \frac{\theta_\alpha^2 s_{\alpha 1}^{2\alpha} (1 - s_{\alpha 1})^{1-2\alpha} E(\theta_\alpha | \theta_\alpha > \theta_\alpha^*)}{2(1 + r_{\alpha 1})}} .$$

Since this expression is always positive, the arguments used to establish the direction of the distortions in the one-loan problem can be

reiterated when analyzing the separating contract parameters of the second stage of the two-loan problem.

As in the single transaction case, both the interest rate and the coefficient of the x input in the type α separating contract exceed the corresponding values of the complete information type α parameters. That is, $\hat{r}_{\alpha 2}(\omega) > \bar{r}_{\alpha 2}$ and $\hat{s}_{\alpha 2}(\omega) > \bar{s}_{\alpha 2}$ when $\beta < \alpha$. However, the magnitude of the distortions depends on the relative likelihoods of the two technologies under the terms of the first-period pooling contract for each value of ω . In fact, the distortions in both parameters increase monotonically with the expression in equation (5.4).

First consider the distortions in the interest rate for the type α firms as the likelihood ratios change. It has already been noted that $\lambda(\omega)$, the Lagrange multiplier, is positive as long as the constraint facing the lender is binding. Furthermore, by rewriting equation (A.10) as

$$(5.5) \quad \lambda(\omega) = \frac{-2[R_{\beta} - C(\bar{r}_{\beta 2}(\omega))]}{[1 - F_{\beta}(\theta_{\beta}^*)]} ,$$

it can easily be determined that $\lambda(\omega)$ is finite since the lender would never allow $\hat{r}_{\beta 2}(\omega)$ to fall below i , the rate the lender must pay to acquire funds. Therefore, the RHS of equation (A.8) approaches zero as $g_{\alpha}(\omega)$ becomes large relative to $g_{\beta}(\omega)$. Hence, we can conclude that $\hat{r}_{\alpha 2}(\omega)$ approaches $\bar{r}_{\alpha 2}$ from above as the likelihood ratio in equation (5.4) decreases.

Not only does the interest rate for a type α firm fall as the likelihood ratio for outcome ω decreases, but the input coefficient

moves closer to the efficient point of production. To see this, subtract equation (A.8) from equation (A.9) to obtain:

$$(5.6) \quad C(\hat{r}_{\alpha 2}) = \lambda(\omega) \frac{g_{\beta}(\omega) A_{\beta}(\hat{s}_{\alpha 2}(\omega))^{2\beta} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\beta}}{g_{\alpha}(\omega) A_{\alpha}(\hat{s}_{\alpha 2}(\omega))^{2\alpha} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\alpha}} \\ \times [1 - F_{\beta}(\theta_{\beta}^*)] \left| \frac{2\beta - 2\alpha}{2\alpha - \hat{s}_{\alpha 2}(\omega)} \right|.$$

A decrease in $\hat{r}_{\alpha 2}(\omega)$ generated by an increase in $g_{\alpha}(\omega)$ increases the LHS of equation (5.6). Noting that $\lambda(\omega)$ is bounded and that the RHS of equation (5.6) is decreasing in the factor share (since $\hat{s}_{\alpha 2}(\omega) > 2\alpha$ for all ω in Ω'), we can conclude that $\hat{s}_{\alpha 2}(\omega)$ must move closer to 2α to preserve the equality. In other words, a firm with type α technology is allowed to produce closer to its productively efficient point as the likelihood ratio diminishes for outcome ω .

We have just determined that the separating contract offered to firms characterized by type α technology moves closer to the complete information contract as the likelihood ratio decreases. The interest rate falls and the factor share moves closer to the cost-minimizing input combination. However, some distortion in the separating parameter values will remain as long as the density functions evaluated at ω are positive for both technology types. Analogous results hold for the firms characterized by type β technology. Under a separating scheme, these firms continue to operate efficiently. In addition, the interest rate charged to them is lower than the complete information type β interest rate as long as both densities are positive for

outcome ω ; but the interest rate increases as $g_\beta(\omega)$ increases relative to $g_\alpha(\omega)$.

The conclusion that the type β firms will never suffer distortions in their factor shares, regardless of the first project outcome, follows directly from equation (A.11). On the other hand, the interest rate for the type β firms with outcome ω will depend on the likelihood ratio. Equation (A.10), which is derived from the first-order condition of the lender's expected profit function with respect to the separating type β interest rate, has the same functional form as equation (4.25) in the one-loan separating problem. The densities do not enter directly into the determination of $\hat{r}_{\beta 2}(\omega)$, but they do enter indirectly through the Lagrange multiplier. The arguments used to analyze the movements in the type α contract parameters can be invoked to demonstrate that the RHS of equation (A.12) must increase as $g_\alpha(\omega)$ increases relative to $g_\beta(\omega)$. The numerator is increasing since $\hat{s}_{\alpha 2}(\omega)$ decreases toward 2α , and therefore toward 2β (recall that $2\beta < 2\alpha < \hat{s}_{\alpha 2}(\omega)$), at the same time that the denominator is decreasing. Because $\hat{s}_{\beta 2}(\omega)$ is independent of the likelihood ratios, the type β interest rate, $\hat{r}_{\beta 2}$, must also decrease in order to continue to assure the satisfaction of the constraint $E\pi_\beta(\hat{s}_{\beta 2}(\omega), \hat{r}_{\beta 2}(\omega)) \geq E\pi_\beta(\hat{s}_{\alpha 2}(\omega), \hat{r}_{\alpha 2}(\omega))$. The reverse argument holds when $g_\beta(\omega)$ grows large relative to $g_\alpha(\omega)$. Hence, we can conclude that the separating type β contract moves closer to the complete information contract intended for type β firms as the likelihood of realizing outcome ω increases for firms characterized by the type β technology.

As the lender becomes more certain that the firms with outcome ω are likely to be of type α rather than type β , the contract parameters are set correspondingly closer to the complete information terms for technology α at the risk of having to pay off type β firms with the same project outcome even more generously. If the reverse mix of firms obtains in the group with first project outcome ω , the lender's optimal strategy involves more gross distortions away from the complete information contract for the type α firms, resulting in a decreased expected profit for both the type α firms and the lender. In return, the lender can charge a higher interest rate to the type β firms and still satisfy the incentive constraint. This strategy allows the lender to economize on the requisite payoff to the relatively large expected number of type β firms with this particular first project return.

Once the decision problem for the second loan is formulated and the second-period contract terms have been determined as functions of the first-period parameters, the lender must optimally choose these first-stage pooling contract terms. In addition to the return from the loans extended in the first stage, the lender must also consider the effects of the pooling contract parameters on the quality of information derived from the first-stage outcome. That is, the lender's choice of the pooling factor share and interest rate affect the density functions, $g_j(\omega)$, and the probabilities, p_j , that enter into the determination of the expected returns in the second period. The lender faces a trade-off between the returns stemming directly from the first-stage

contracts and the quality of information available in determining the second-stage contract terms. Even though the lender makes no promises to anyone and faces no constraints of any kind in constructing the initial loan contract, the choice of the parameters for the first loan do influence the expected returns from the second-period loans.

Just as in the one-loan model, the lender determines its optimal choice of contract parameters by summing its expected return from loans to firms of both types of technologies, weighted by the number of such firms in the market. In addition, the lender now also sums over the loans made in both stages. Formally, the lender's perfect two-loan maximization problem, given that it has chosen to pool all loan applicants for the first loan, can be set forth as follows:

$$\begin{aligned}
 (5.7) \quad E_p = \max \quad & \sum_{j=\alpha, \beta} [N_j E p_j(s_{p1}, r_{p1}) + p_j N_j E p_j(\bar{s}_{j2}, \bar{r}_{j2})] \\
 & + \int_{\Omega} \left\{ \sum_{j=\alpha, \beta} g_j(\omega) N_j E p_j(s_{j2}(\omega), r_{j2}(\omega)) \right. \\
 & \left. + \lambda(\omega) g_{\beta}(\omega) N_{\beta} [E \pi_{\beta}(s_{\beta 2}(\omega), r_{\beta 2}(\omega)) - E \pi_{\beta}(s_{\alpha 2}(\omega), r_{\alpha 2}(\omega))] d\omega \right\} .
 \end{aligned}$$

The expectations operator is used with respect to the random project returns; the probabilities p_{α} and p_{β} enter as part of the determination of the lender's overall expected return. $\lambda(\omega)$ is the Lagrange multiplier associated with the type β incentive constraint for firms with project outcome ω . The lender must maximize equation (5.7) over s_{p1} and r_{p1} , taking account of the effect of these variables on the contract terms offered in the second stage. The system of equations generated by the first-order conditions is set out in Appendix A.

Without assumptions on the form of the distribution functions $F_{\alpha}(\theta_{\alpha})$ and $F_{\beta}(\theta_{\alpha})$, it is difficult to arrive at definitive conclusions about the properties of the first-stage pooling contract relative to the complete information contracts or the optimal pooling contract that obtains when only a single transaction is considered. The difficulties arise because irregularities in the density function have not been ruled out; consequently, the impact of the first-period contract parameters on the distribution of second-period returns cannot be determined. However, if the distributions are assumed to be uniform, for example, results similar to those drawn from the one-loan model can be derived. The discussion through the remainder of this section will maintain the assumption of uniform densities for both technology types.

When the returns to both types of firms are distributed uniformly, the qualitative results of the single transaction pooling contract carry over to the first stage of the two-loan problem. The interest rate assumes an intermediate value between the complete information interest rates of the two technology types. The factor share in the pooling contract will lie between the efficient points of the two types of firms in some circumstances but not in others.

The calculations needed to determine the form of the first-period contract are greatly facilitated by the simplifications permitted by the uniform distribution. The likelihood ratios for all project outcomes in the region Ω' are identical. Consequently, the lender's maximand, equation (5.7) can be rewritten as^{14/}

$$\begin{aligned}
 (5.8) \quad E\rho = \max_{j=\alpha,\beta} \sum_j [N_j E\rho_j(s_{j1}, r_{j1}) + p_j N_j E\rho_j(\bar{s}_{j2}, \bar{r}_{j2}) \\
 + (1 - p_j) N_j E\rho_j(s_{j2}, r_{j2})] \\
 + \lambda(1 - p_\beta) N_\beta [E\pi_\beta(s_{\beta2}, r_{\beta2}) - E\pi_\beta(s_{\alpha2}, r_{\alpha2})] .
 \end{aligned}$$

Note further that the probability p_j is independent of \hat{r}_{j1} . This claim can be supported by evaluating the probability of positive identification after changing variables from ω , the outcome observed by the lender, to θ_j . This will be carried out for technology α ; a similar argument can be used to establish the same property with respect to p_β .

The probability that a firm with technology α will be unambiguously identified by its first-stage outcome, assuming that $F_\alpha(\theta_\alpha)$ is uniform with support $[\theta_\alpha, \bar{\theta}_\alpha]$, can be written as

$$(5.9) \quad p_\alpha = \int_{\hat{\theta}_\alpha}^{\bar{\theta}_\alpha} f_\alpha(\theta_\alpha) d\theta_\alpha ,$$

where $\hat{\theta}_\alpha$ is defined by the equation

$$\begin{aligned}
 (5.10) \quad \hat{\theta}_\alpha &= \frac{\gamma_\alpha^{2\hat{s}_{\alpha1}} (1 - \hat{s}_{\alpha1})^{1-2\alpha} E(\theta_\alpha | \theta_\alpha > \theta_\alpha^*)}{2(1 + \hat{r}_{\alpha1})} \\
 &= \frac{\gamma_\beta^{2\hat{s}_{\beta1}} (1 - \hat{s}_{\beta1})^{1-2\beta} E(\theta_\beta | \theta_\beta > \theta_\beta^*)}{2(1 + \hat{r}_{\beta1})} .
 \end{aligned}$$

The interest rate only scales the size of the project outcome; it serves no role in identifying the value of θ_α that equates the type α outcome with the outcome attained by a type β firm at the upper endpoint of the type β support. Hence, it can be concluded that p_α is independent of the interest rate.

To see that the first-stage pooling interest rate lies between \bar{r}_α and \bar{r}_β , consider the first-order condition with respect to the first-stage pooling interest rate:

$$(5.11) \quad C(\hat{r}_{pl}) = \sum_{j=\alpha,\beta} \frac{w_j}{w_\alpha + w_\beta} R_j ,$$

where

$$(5.12) \quad w_j = N_j A_j (\hat{s}_{pl})^{2j} (1 - \hat{s}_{pl})^{1-2j} , \quad j = \alpha, \beta .$$

As is easily verified, equation (5.11) is exactly the same as equation (4.33). Hence, the proof that $\bar{r}_{\alpha 1} < \hat{r}_{pl} < \bar{r}_{\beta 1}$ follows without alteration from the one-loan model.

When the lender chooses to pool at the first stage of the two-loan model, again assuming uniform distributions for project outcomes, the optimal value of the input share is even more likely to exceed 2α than it was in the single transaction case. This conclusion can be derived from examination of the first-order condition of equation (5.8) with respect to the factor share, s_{pl} :^{15/}

$$(5.13) \quad \frac{2\alpha - \hat{s}_{p1}}{2\beta - s_{p1}} = \frac{N_{\beta} E\rho_{\beta}(s_{p1}, r_{p1}) - N_{\alpha} T_{\alpha}(\bar{\omega}) - N_{\beta} T_{\beta}(\underline{\omega})}{N_{\alpha} E\rho_{\alpha}(s_{p1}, r_{p1}) + N_{\alpha} T_{\alpha}(\bar{\omega}) + N_{\beta} T_{\beta}(\underline{\omega})} ,$$

where

$$(5.14) \quad T_j(\omega) = g_j(\omega) \omega [E\rho_j(\bar{s}_{j2}, \bar{r}_{j2}) - E\rho_j(\hat{s}_{j2}, \hat{r}_{j2})] , \quad j = \alpha, \beta .$$

Once again, the lender can be assured of positive expected returns from the loans extended to the type α firms. Hence, the denominator on the RHS of equation (5.13) is always positive. The expression in braces in the numerator is positive if and only if the lender's expected return from loans granted to the type β firms are positive and large enough to satisfy the inequality

$$(5.15) \quad N_{\beta} E\rho_{\beta}(s_{p1}, r_{p1}) > N_{\alpha} T_{\alpha}(\bar{\omega}) + N_{\beta} T_{\beta}(\bar{\omega}) .$$

Note that the RHS of this inequality is always non-negative since the lender's expected return from any firm is maximized under the terms of the complete information contract. When inequality (5.15) holds, the first-stage optimal pooling input coefficient lies between 2α and 2β . When the inequality is reversed, however, both types of firm are forced to overinvest in the x input. The overinvestment is a more likely outcome when repetition of the transactions is anticipated because the improvement in the information available in the second stage compensates for some of the loss from the inefficiencies of the first-stage loan contract parameters.

In brief, the properties of the loan contract parameters that obtain when the lender chooses to pool for the first stage of a two-loan problem can be described as follows:

The pooling interest rate in the first stage of a two-loan problem is exactly the same as in the one-loan pooling problem when the returns to both types of firms are distributed uniformly. The optimal input share either lies between the efficient points for the respective technologies, or it exceeds both of them.

In the second stage, complete information contracts are offered to those firms that are unambiguously identified by their first project return. All other firms are offered separating contracts, the terms of which are contingent upon their first-period project returns. The distortions in the separating contract parameters are qualitatively the same as in the one-loan separating problem. However, their magnitude is monotonic in the likelihood ratio. The separating contracts move closer to the complete information contract for the technology type most likely to realize outcome ω . The results pertaining to the second-period parameters are independent of the distributional assumptions on the random project returns.

Because of the high cost of separating firms at the outset when perfection is imposed on the lender's strategies, pooling contracts in the first period are no longer dominated. Of course, only separating contracts are offered to unidentified borrowers at the start of the second period.

6. Imperfect Strategies in a Two-Period Model with Loan Size Variable

If the lender's feasible strategies need not be perfect, the lender can make threats and promises in one stage of negotiations that bind its actions in another. The lender is no longer forced to maximize

its remaining expected returns at each stage of contracting. The increased flexibility in the lender's strategy choices makes it easier to maintain standing relationships with the loan customers that do have positive value for the lender beyond the acquisition of improved information. The assumption that strategies need not be perfect is reasonable in contexts in which the lender's reputation for adhering to its bargains plays a role. For example, word-of-mouth communication among entrepreneurs is a vital conduit for ascertaining information about venture capitalists. In this manner, the venture capitalist's dealings with one firm can influence subsequent negotiations with other firms.

6.1 Separating Contracts for the First Loans

Imperfect strategies allow the lender to elicit the cooperation of the type β firms by offering believable promises of lenient contract terms for one or both loans in return for sharing the knowledge of the true nature of the technology type at the outset of the negotiations. If the lender must pay off the type β firms in order to elicit their cooperation, it might be profitable to distribute these rewards over two loans rather than concentrate them all in just one contract; hence, the need for promises. The lender can also threaten to refuse funds completely at the second stage if the firm is suspected of giving false information. Such drastic threats make cooperation a more attractive alternative for the type β firms relative to the possibility of revelation of misrepresentation, thereby increasing the expected returns of the lender.

Several matters must be considered before one can even attempt to formulate the problem when the lender has decided to separate at the outset of a two-loan problem in which imperfect strategies are permitted. For example, the form of the optimal threat for failure to cooperate must be studied. In addition, one must decide which of the second loan contracts should be dependent on the project outcomes from the first loans.

One aspect of the imperfect separating equilibrium policy can be determined a priori. Refusal of a second loan to any firm that has unambiguously lied about its technology when applying for the first loan is the lender's optimal threat. By definition, in a separating equilibrium each firm ultimately accepts the loan contract intended for its type of technology. Therefore, the lender's threat to potentially dishonest type β firms enters only as a deterrence mechanism. The deterrence is maximized by threatening the worst possible outcome for the firm, complete refusal to extend funds.

We can also establish the property that the contract terms offered to the type β firms should never depend on the first project outcomes even in the second stage of transactions. The lender's second-period return from type β firms depends on the distributions of θ_β , but it is independent of the realization of the project outcome in the first period. Therefore, given that the lender has determined the identity of the firms through the separation process in the first period, the contract terms offered to any type β firm in the second stage should not depend upon the density functions evaluated at any particular outcome.

Type α firms' contract terms in the second stage, however, must still be set in such a way that deters type β firms from taking the type α contract, given the information the lender has acquired from the returns to the projects financed by the first loans. Therefore, the likelihood of each of the respective firms obtaining the particular return under consideration may enter into the determination of the contract terms of the second loan extended to type α firms.

With all of these considerations in mind, the lender's maximand for an imperfect strategy in which separation is sought from the outset can now be addressed. The expected returns generated from each type of technology from the first separating loans are weighted by the respective number of firms in the market characterized by that particular technology. Because each firm's technology has been identified by separation from the beginning, all type β firms are unconditionally promised the same contract terms for the second loan. Remembering that the second type α contract must still play a deterrence role, the remaining type α firms will be granted contract terms that are conditional on the outcome of their first investment projects. These contract parameters must be set to satisfy the incentive constraint so that the type β firms will indeed find their own contracts more profitable than the type α contracts. The lender maximizes the following function over the entire set of policy variables $(s_{\alpha 1}, r_{\alpha 1})$, $(s_{\beta 1}, r_{\beta 1})$, $(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2})$, $(s_{\alpha 2}(\omega), r_{\alpha 2}(\omega))$, $(s_{\beta 2}, r_{\beta 2})$ and the Lagrange multiplier λ :

$$\begin{aligned}
 (6.1) \quad E\rho = \max_{\alpha} & N_{\alpha} \{ E\rho_{\alpha}(s_{\alpha 1}, r_{\alpha 1}) + p_{\alpha} E\rho_{\alpha}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2}) \\
 & + \int_{\Omega} g_{\alpha}(\omega) E\rho_{\alpha}(s_{\alpha 2}(\omega)) d\omega \} + N_{\beta} \{ E\rho_{\beta}(s_{\beta 1}, r_{\beta 1}) \\
 & + E\rho_{\beta}(s_{\beta 2}, r_{\beta 2}) + \lambda [E\pi_{\beta}(s_{\beta 1}, r_{\beta 1}) \\
 & + E\pi_{\beta}(s_{\beta 2}, r_{\beta 2}) - E\pi_{\beta}(s_{\alpha 1}, r_{\alpha 1}) \\
 & - p_{\beta} \cdot 0 - \int_{\Omega} g_{\beta}(\omega) E\pi_{\beta}(s_{\alpha 2}(\omega), r_{\alpha 2}(\omega)) d\omega] \} .
 \end{aligned}$$

The first-order conditions derived from this maximand appear in Appendix B.

Once again, we can use dynamic programming to study the expected returns to the lender and the properties of the contracts offered to the firms. Starting with the second stage, the type α contracts can be determined conditional on the realization of the returns from the first loans. Then the lender can proceed to choose the optimal first-stage contracts since we argued above that the form of the second loan contract offered to firms with type β technology would not depend on any outcomes realized in the first stage.

The analysis of the second loan contract offered to type α firms carries over from the one-loan model with minimal alteration. Those firms that are unambiguously identified at the start of the second stage are offered their complete information contracts. All other type α firms suffer some degree of distortion in their separating contract parameters. Just as in the case of perfect strategies, the magnitude of the distortions in the type α separating contracts is monotonically

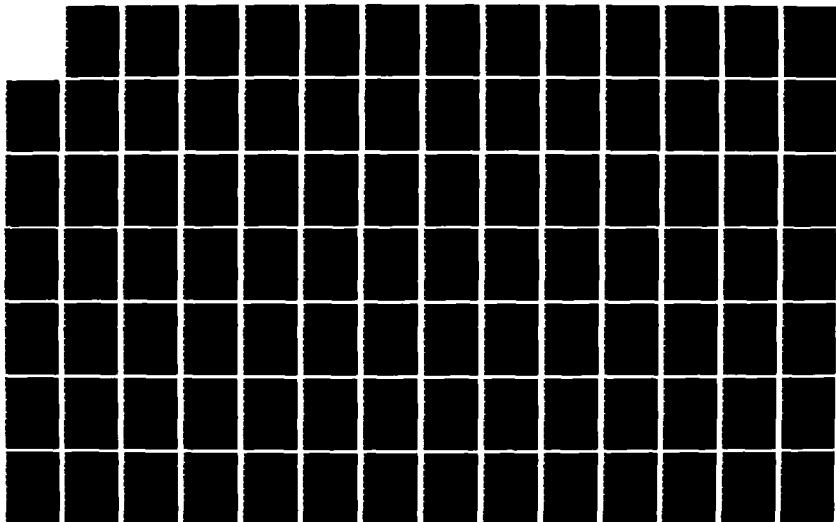
AD-A163 192

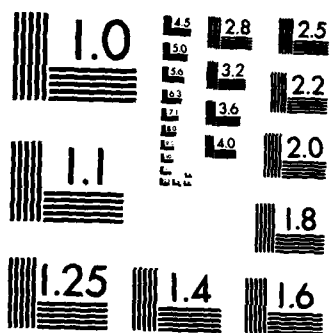
INTERACTIONS BETWEEN SIGNALING AND REPEATED PLAY WITH
BORROWER DEFAULT(U) STANFORD UNIV CA INST FOR
MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES C L SUCH
OCT 85 TR-480 N00014-79-C-0685 F/G 5/3

273

UNCLASSIFIED

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

decreasing in the likelihood ratio evaluated under the terms of the first-period type α contract,

$$(6.2) \quad \frac{g_{\beta}(\omega)}{g_{\alpha}(\omega)} = \frac{g_{\beta} \left[\theta_{\beta} \frac{\gamma_{\beta}^2 s_{\alpha 1}^{2\beta} (1 - s_{\alpha 1})^{1-2\beta} E(\theta_{\beta} | \theta_{\beta} > \theta_{\beta}^*)}{2(1 + r_{\alpha 1})} \right]}{g_{\alpha} \left[\theta_{\alpha} \frac{\gamma_{\alpha}^2 s_{\alpha 1}^{2\alpha} (1 - s_{\alpha 1})^{1-2\alpha} E(\theta_{\alpha} | \theta_{\alpha} > \theta_{\alpha}^*)}{2(1 + r_{\alpha 1})} \right]} .$$

When determining the parameters of the contracts for the first loans, the lender must again trade off expected returns in the first stage for informational improvement in the second stage. However, only the α contract parameters affect the quality of information available in the second stage. If a firm selects a type β contract at the outset, the lender learns no more about the firm by observing its project outcome since the firm's contract selection is sufficient to reveal its true technology type. The lender need only be concerned with its ability to distinguish returns generated by true type α firms from those that might have been produced by type β technology under the terms of a type α contract.

If the distribution functions of the project returns for both technology types are sufficiently well-behaved, the qualitative properties of the loans extended to type α firms in the first stage are the same as in the one-loan model. However, determinate conclusions are difficult to establish without specific distributional assumptions. Therefore, the first-period contracts extended to type α firms will be analyzed under the assumption that both $F_{\alpha}(\theta_{\alpha})$ and $F_{\beta}(\theta_{\beta})$ are uniformly distributed.

All firms with type α technology will have to pay a higher interest rate for their first loans than the complete information rate. As we noted in Section 5, the lender's expected return from perfect pooling strategies can be written in terms of the probabilities, p_j , without reference to the density functions, $g_j(\omega)$, when the returns to both types of firms are distributed uniformly. Since $r_{\alpha 1}$ does not affect the probabilities, the first-order condition with respect to the first-stage type α separating interest rate is identical in form to the corresponding condition in the one-loan problem. That is, differentiation of equation (6.1) with respect to $r_{\alpha 1}$ yields

$$(6.3) \quad [R_{\alpha} - C(\hat{r}_{\alpha 1})] = \frac{\lambda}{2} \frac{N_{\beta} A_{\beta} \hat{s}_{\alpha 1}^{2\beta} (1 - \hat{s}_{\alpha 1})^{1-2\beta}}{N_{\alpha} A_{\alpha} \hat{s}_{\alpha 1}^{2\alpha} (1 - \hat{s}_{\alpha 1})^{1-2\alpha}} [1 - F_{\beta}(\theta_{\beta}^*)] .$$

Using the same argument as before, equation (6.3) establishes the result that $\hat{r}_{\alpha 1} > \bar{r}_{\alpha 1}$ for all type α firms. However, the size of the distortions cannot be compared to that of the one-loan model since the contract parameters are evaluated at different values.

Overinvestment in the x input occurs for all type α firms in the first stage under the assumption of uniform distributions. The same methods of proof as in the one-loan model can be implemented here. The differentiation of the lender's expected return over both loans with respect to $s_{\alpha 1}$ leads to:

$$\begin{aligned}
 (6.4) \quad & N_{\alpha} [E\rho_{\alpha}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + T_{\alpha}(\bar{\omega})] + \lambda N_{\beta} g_{\beta}(\omega) \omega E\pi_{\beta}(\hat{s}_{\alpha 2}, \hat{r}_{\alpha 2}) \\
 & = \lambda N_{\beta} [E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + g_{\beta}(\omega) \omega E\pi_{\beta}(\hat{s}_{\alpha 2}, \hat{r}_{\alpha 2})] + N_{\alpha} T_{\alpha}(\bar{\omega}) \left[\frac{2\beta - \hat{s}_{\alpha 1}}{2\alpha - \hat{s}_{\alpha 1}} \right],
 \end{aligned}$$

where $T_{\alpha}(\bar{\omega})$ is defined in equation (5.14). Subtracting equation (6.3) from equation (6.4) results in

$$\begin{aligned}
 (6.5) \quad & \frac{1}{2} N_{\alpha} A_{\alpha} C(\hat{r}_{\alpha 1}) \hat{s}_{\alpha 1}^{2\alpha} (1 - \hat{s}_{\alpha 1})^{1-2\alpha} \\
 & = \{ \lambda N_{\beta} [E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + g_{\beta}(\omega) \omega E\pi_{\beta}(\hat{s}_{\alpha 2}, \hat{r}_{\alpha 2})] + N_{\alpha} T_{\alpha}(\bar{\omega}) \} \left[\frac{2\beta - 2\alpha}{2\alpha - \hat{s}_{\alpha 1}} \right].
 \end{aligned}$$

This condition proves that $\hat{s}_{\alpha 1}$ is greater than 2α . The LHS and the term in braces on the RHS of equation (6.5) are both positive. Since $\beta < \alpha$, we can conclude that $2\alpha < \hat{s}_{\alpha 1}$. However, a comparison of the productive inefficiency in the one-loan model relative to the first stage of a two-loan model is ambiguous.

In the two-loan model, we can demonstrate the same qualitative results for both loans extended to type β firms as were established in the one-loan separating problem. These conclusions are stronger than the ones with respect to the type α contracts; they are independent of any distributional assumptions since no second-period type β contract depends on first-period project realizations. These firms are allowed to select their cost-minimizing input combinations in both stages. Moreover, they are rewarded with an interest rate lower than the complete information rate for both loans in exchange for cooperation in

revealing their true technology type. The analysis from the one-loan problem can be applied here without alteration. The lender optimally distributes the reward of the lower interest rate over both loans. This follows from noting that the first-order conditions with respect to $\hat{r}_{\beta 1}$ and $\hat{r}_{\beta 2}$, equations (B.8) and (B.10), respectively, are the same function. The type β interest rates enter the lender's maximand symmetrically; furthermore, the lender's profit function is concave in the interest rates. Therefore, the lender's optimal contract offering set $\hat{r}_{\beta 1}$ equal to $\hat{r}_{\beta 2}$.

The properties of the two-loan model when imperfect strategies are feasible for the lender can be summarized as follows:

When the lender offers separating contracts at the outset of the two-loan model with imperfect strategies, all type α firms must overinvest in the input used as a signal and pay a higher interest rate than the complete information rate in the first stage (under the assumption of uniform distributions of the project returns). Those that are unambiguously identified at the start of the second stage will receive their complete information contract for the second loan. All others will suffer the same sort of distortions as in the first stage, but the magnitude of the distortions will decrease with the likelihood ratio for the type α technology. The type β contracts for both loans are identical, regardless of the first project. These firms are always allowed to produce efficiently; in addition, they are charged a lower interest rate than the complete information rate in both stages. The results with respect to the type β contracts hold without the need to resort to specific distributional assumptions.

Unlike perfect strategies, imperfect strategies guarantee the lender no loss in expected returns from the maintenance of standing relationships with its borrowers (relative to single-loan contracts). In most instances, such alliances have positive value for the lender.

For example, if all firms would inevitably be identified by returns to their first projects, the threat of withholding funds for a second loan would facilitate separation of the firms at the outset. Not only does the lender learn the identities of the firms with certainty before extending the second loans, but the potent deterrence mechanism allows it to significantly increase its expected return from the first loans. The lender's gain over and above the benefits derived from improvements in information could be substantial. Strong commitments enhance the overall efficiency of the credit market because they help alleviate the negative impact of the externalities caused by the potentially dishonest type β firms. It is noteworthy that market power on the part of the lender is prerequisite to the credibility and influence of these commitments.

6.2 Pooling Contracts for the First Loans

Pooling contracts can be ruled out even when multiple transactions are anticipated, provided that the lender is allowed to follow imperfect strategies. To understand this conclusion, suppose that the lender chooses to pool in the first period. When pooling contracts are offered, the lender's strategy entails no commitments that bind the terms of the second loans. Hence, optimal separating contracts will be offered in the second stage to all unidentified firms. The lender could accomplish exactly the same results in the second period but improve its expected return from the loans offered in the first period. Just as in the one-loan problem, the pooling contract almost always involves inherent productive inefficiencies for both technology types. These

inefficiencies could be reduced. Some pair of separating contracts could be constructed that provided the same expected profits for each type of firm but improved the expected returns for the lender. Since all forms of commitments are permitted under imperfection, the lender's second-period decisions need not be bound by any of the information gained from the first period. Therefore, the lender strictly improves its total expected returns over the two-loan horizon by separating firms at both stages.

7. Introduction to the Fixed-Loan Size Model

The market under investigation is one in which small, start-up firms are dependent on a single lender for production loans. Each entrepreneur possesses private information about his potential as a small businessman and the risk involved with the available technology. This private information is correlated with observable features of the operation of the business enterprise.

Much of the negotiation between the contracting parties centers on the business plan. The lender's influence over these decisions reflects its experience with past borrowers and its notion of sound, money-making enterprises. The terms of the plan significantly affect the expected profits of both parties. Therefore, the lender retains some power in setting these terms.

Potential borrowers differ in the degree of uncertainty about the actual quality or sales price of their final output and, ultimately, in the probability of loan repayment. Each entrepreneur has just developed

plans for a new, high-technology product. Some of these projects are primarily of engineering interest. Other borrowers have more business acumen and are more likely to produce highly marketable output. Ideally, the lender would like to be able to distinguish these two groups of entrepreneurs and maximize its expected return by offering different contract terms to each, commensurate with the risk involved.

In fact, the lender can only distinguish firms imperfectly. Much of the formal business plan is devoted to the allocation of time committed to various business functions. Consequently, the lender can observe each borrower's intended division of time between, for example, administrative and engineering activities. It is assumed that the optimal time allocation is correlated with the degree of risk across firms. Both the efficient time allocation and the risk of project outcome are intrinsic to the firm and unalterable throughout the problem. Presumably, the lender can infer that those entrepreneurs who plan relatively intensive concentration on administrative duties are likely to be the ones with the highly marketable products and low risk of loan default.

The operating premise of the model is that the lender can more accurately assess the true entrepreneurial skill of a potential borrower by the business plan than it can from the actual dollar amount requested. Attention will be focused on the particular specification of these plans. Credit rationing will not arise as an issue. Specific features of the model will make it technologically efficient for all firms to borrow the same dollar amount. The fixed loan size is the key distinction in the modeling. No private information can be transmitted through

the size of the loan request; therefore, the lender gains nothing by rationing credit and is justified in ignoring the issue.

It is assumed that the firm borrows funds to buy raw materials, and the purchase price of the productive inputs can be documented. Since this information does not vary among firms with different technology types, the lender is precluded from acquiring pertinent information about a firm's identity solely on the basis of the size of the loan demand. Not only can the loan size be documented, but it is also assumed that the total amount of funds requested is the same for all firms in the market.

Two aspects of risk are associated with the projects undertaken by each entrepreneur. There is some probability that the project will collapse entirely, yielding a final output of zero. Since these projects are innovative and somewhat experimental, no entrepreneur can eliminate this possibility entirely; but each can exert some influence over the odds through the efficient division of work time. This allows the lender to use specifications of the work plan as a signaling mechanism because the time allocation has a direct effect on firm profitability.

Even given that some tangible final output will be produced, however, the quality of that output is still random. This portion of the risk is exogenous. It might stem from uncertainty about the quality of the raw materials, elements of the environment that affect production, or reception of the product in the final market. The lender attempts to set an interest rate that compensates for the actual risk of

return. Even if all technological information were freely available to the lender, a different interest rate would be charged to each type of firm because of the differences in the exogenous portion of the risk. This is the motivating force behind the lender's attempts to categorize firms by their technologies.

All firms characterized by a particular technology are identical with respect to their productive opportunities and the risk at stake. However, each entrepreneur has some reservation wage for his small business activities; and these opportunity costs vary across firms. The reservation wage can be thought of as the entrepreneur's valuation of working for a large corporation rather than pursuing his small business enterprise. Therefore, the aggregate loan demand function is downward-sloping even though each individual loan is fixed in size. The contract terms offered by the lender cannot influence the size of the loan request, but they do affect the total volume of loan demand. Each entrepreneur has some alternate activity which he would choose to do if the expected profit from his small business venture were too small under the lender's established contract terms.

The lender might want to design contracts that exploit the differences among entrepreneurs within a single technological category. For example, it might be desirable to induce those entrepreneurs with high reservation wages to identify themselves so that a higher interest rate can be charged to the remaining borrowers. Although knowledge of these personal differences could be profitable to the lender, it will be shown that it is impossible to extract this information in a single

transaction or in repeated negotiations restricted by perfection. Only when the flexibility of imperfect strategies is permitted can the lender take advantage of the variation in reservation wages among firms of the same technological type.

8. One-Period Model with Loan Size Fixed

Initially, a model will be considered in which only a single transaction is negotiated between the lender and any given firm. Properties of the optimal contracts will be analyzed under the assumption that the lender costlessly shares all information about each firm's underlying technological capabilities. These complete information contracts will serve as a base case for purposes of comparison with those contracts devised under the more realistic asymmetric information conditions. Both pooling and separating contracts will be analyzed. In the one-loan model, it will be demonstrated that pooling strategies are always dominated. Nevertheless, it will be useful to have characterized the optimal pooling contract in the single-loan model for use in more complicated contexts.

All agents in the market maximize expected profits. The borrowers have private information about their technological potential and their individual opportunity costs. The lender knows the true distribution of these pieces of data but cannot freely categorize a particular firm nor ascertain an entrepreneur's reservation wage. The lender uses its limited information to construct contracts that maximize its expected returns, taking into account the expected number of firms in the market

under any given set of contract terms. A potential entrepreneur then calculates its expected profit under each contract and chooses the one that yields the highest result. Note that an entrepreneur's acceptance of a particular set of contract terms could partially reveal some of the firm's private information.

The size of each firm's project is fixed exogenously and is normalized to one. This formulation eliminates the opportunity for the lender to gain information about any firm's identity through the size of its loan request. Thus, credit rationing in the sense of restricting the amount of funds lent to any one individual is irrelevant in this model.

Each loan applicant belongs to one of two groups of firms characterized by their unalterable technological potential. The two technology types differ in the productively efficient division of time spent on various business activities and in the degree of risk inherent in their investment projects. The low-risk firms are identified as type α firms; the high-risk borrowers are type β firms. By assumption, $\alpha, \beta \in (0,1)$ and $\alpha > \beta$.

Each firm faces some inevitable risk of producing no salable goods. The probability of positive output is a function of the division of the entrepreneur's time among his business activities. Since this division of time is an integral part of the business plan, the lender can influence the firm's behavior by specifying a particular time allocation as part of the contract. In particular, the lender might

manipulate this instrument in an attempt to acquire information about the firm's technology.

Given that output is positive, the quality of the final product is still random. The conditional distribution of θ_j , the quality of the output for technology j ($j = \alpha, \beta$), is exogenous and identical for all firms within a technological category. The support of θ_j is connected; that is, there are no "gaps" in the potential level of realized quality.

Each firm in the market is entirely dependent on debt financing for the project expenses. It secures a loan of size one and repays it at a rate of $(1 + r_j)$ if the revenue from the sales is sufficiently large to cover the debt. If revenues are inadequate to repay the entire obligation, all proceeds from the project are awarded to the lender. Hence, the realized profit for any given firm of type j is

$$(8.1) \quad \pi_j = \max\{0, \theta_j - (1 + r_j)\}, \quad j = \alpha, \beta.$$

Each entrepreneur bases his decision to apply for a loan on his expected profit as a small businessman relative to his reservation wage. Expected profit depends on the probability of positive output and the conditional distribution of quality, given positive output. The former is a function of the entrepreneur's division of time between two activities and is denoted by $p_j(s)$, $j = \alpha, \beta$. It is assumed that p_j is concave in s , the fraction of the work period devoted to the first activity, and attains its maximum value at $s = j$. Therefore,

the optimal allocation of time from the firm's point of view is

$$s_j = j \quad (j = \alpha, \beta).$$

It is also assumed that, for a fixed interest rate, $\frac{\partial E\pi_\alpha(s,r)}{\partial s} > \frac{\partial E\pi_\beta(s,r)}{\partial s}$ for all values of s .^{16/} This inequality insures that the second-order conditions hold globally. It imposes a regularity condition on the shape of the firms' isoprofit curves; it is a sufficient condition for the separation results that follow.

Under the foregoing assumptions, the firm's expected profit function can now be written as

$$(8.2) \quad E\pi_j(s,r) = p_j(s) \int_{1+r}^{\infty} [\theta_j - (1+r)] f_j(\theta_j) d\theta_j, \quad j = \alpha, \beta,$$

where $f_j(\theta_j)$ is the probability density function of the quality variable, θ_j . The entrepreneur decides to proceed with his business venture if $E\pi_j(s,r)$, evaluated under any of the contract terms offered by the lender, exceeds the entrepreneur's reservation wage, indexed by γ_j . Although the loan demand from any one borrower is fixed in size, the variation in the entrepreneurs' reservation wages causes the aggregate loan demand to be an increasing function of the expected profit for an individual firm.

The lender receives the entire return to the project up to a ceiling of $(1+r)$. It pays a constant market rate of $(1+i)$ for its loanable funds. The lender's realized return from a single firm of type j is

$$(8.3) \quad \rho_j = \min\{\theta_j, (1+r)\} - (1+i), \quad j = \alpha, \beta.$$

The expected return to the lender from an individual firm characterized by technology j is then:

$$(8.4) \quad E p_j(s, r) \\ = p_j(s) \left[\int_0^{1+r} \theta_j f_j(\theta_j) + \int_{1+r}^{\infty} (1+r) f_j(\theta_j) d\theta_j \right] - (1+i), \quad j = \alpha, \beta.$$

By assumption, $E p_j(s, r)$ is a concave function of its arguments.

The lender knows the distribution of the reservation wages, $G_j(\gamma_j)$, for both types of firms.^{17/} (Note that $G_j(\gamma_j)$ can be an arbitrary measure rather than a probability measure.) Therefore, it knows the expected number of each type of firm in the market under any given set of contract parameters. This knowledge is used in constructing the loan contracts so as to maximize the lender's overall expected return.

8.1 Complete Information Case

The lender's optimal contracts in an environment of complete information will again be used as the base case for purposes of comparison with those contracts that obtain when the ex ante information is asymmetric. In this subsection only, it is assumed that the lender can costlessly identify each firm's technological type. Knowledge of an individual entrepreneur's exact reservation wage is irrelevant since all firms of a particular type that choose to be in the market have exactly the same productive potential. The lender's expected return depends on

the total number of borrowers of each technology type, so the only pertinent reservation wage is that of the marginal borrower.

The lender selects a pair of contract terms, (s_j, r_j) , to maximize its total expected return from firms of type j , realizing that the choice of contract parameters delimits the number of entrepreneurs who choose to borrow, Q_j . As the contract specifications become more attractive in terms of the firms' expected profits, a greater number of potential entrepreneurs choose to become small businessmen. Therefore, the lender's problem can be stated as

$$(8.5) \quad E_p = \max_{\substack{s_\alpha, r_\alpha \\ s_\beta, r_\beta \\ Q_\alpha, Q_{\beta 1}}} \sum_{j=\alpha, \beta} \{ p_j(s_j) \left[\int_0^{1+r_j} \theta_j f_j(\theta_j) d\theta_j + \int_{1+r_j}^{\infty} (1+r_j) f_j(\theta_j) d\theta_j \right] - (1+r_j)Q_j + \mu_j [G_j(\gamma_j^*) - Q_j] \} ,$$

where $G_j(\gamma_j^*)$ is the measure of type j entrepreneurs who actually borrow and γ_j^* is given by

$$(8.6) \quad \gamma_j^* = p_j(s_j) \int_{1+r_j}^{\infty} [\theta_j - (1+r_j)] f_j(\theta_j) d\theta_j , \quad j = \alpha, \beta .$$

Note that γ_j^* is the expected profit for any type j firm.^{18/} The lender's expected return is constrained by the need to make expected firm profit large enough to ensure that Q_j entrepreneurs actually do choose to secure loans.

When the lender can obtain complete information, the allocation of time maximizes productive efficiency. That is, $\bar{s}_j = j$ ($j = \alpha, \beta$). (Bars denote the optimal complete information contract parameters.) Since both parties to the loan contract have access to the same information, no controversy can arise over the division of entrepreneurial time among work activities.

The first-order condition of the lender's profit function with respect to the interest rate charged on the loan, r_j , can be written as

$$(8.7) \quad \frac{G_j(\gamma_j^*)}{g_j(\gamma_j^*)} = E\rho_j(\bar{s}_j, \bar{r}_j) \quad , \quad j = \alpha, \beta$$

where $g_j(\gamma_j)$ is the density function of $G_j(\gamma_j)$. The higher is the interest rate, the higher is the lender's return from an individual firm but the lower is the total number of type j firms in the market. Equation (8.7) determines the optimal trade-off between these two considerations.

In brief, the complete information contracts can be characterized as follows:

Each complete information contract specifies a productively efficient time share since this serves the interests of both the lender and the borrowers. The interest rates are set at levels that balance the lender's return per loan extended with the number of borrowers of each technological category.

8.2 Asymmetric Information

If each borrower's identity could be freely determined, the lender would offer contracts that allow for productive efficiency and equate

marginal lender return to the ratio of total to marginal borrowers. These results might not hold when the technological character of the firm is privately held information. Furthermore, it might not be possible for all firms to credibly transmit this information to the lender, even though both parties would benefit from sharing. That is, if the type α loan contract proved to be the more profitable one for both type α and type β firms, all firms would claim to have type α technology. The complete information contracts would fail to induce honest disclosure on the part of all borrowers.

For the rest of the analysis, it is assumed that firms characterized by type β technology find the complete information type α contract more profitable than their own complete information contract. Recall that loan contracts comprise both the interest rate and the allocation of time between two business activities. Furthermore, complete information contracts always specify productively efficient use of time. Therefore, $E\pi_{\beta}(\bar{s}_{\alpha}, \bar{r}_{\alpha}) > E\pi_{\beta}(\bar{s}_{\beta}, \bar{r}_{\beta})$ implies that \bar{r}_{α} is sufficiently lower than \bar{r}_{β} to more than compensate the type β entrepreneurs for their suboptimal use of time under the α contract.

The externality in the market can flow in only one direction in the single-loan model. This result is immediate. Since the type α contract is productively efficient for firms with type α technology and \bar{r}_{α} is lower than \bar{r}_{β} , no firm with type α technology could prefer the complete information type β contract.

It has been assumed that the lender is unable to accurately categorize firms by simply offering complete information contracts for each technology type. Hence, the lender has a choice of two options. It can construct different contracts for each technology type in a way that assures precise classification of the firms by their technology type. Such a scheme must guarantee that the contract intended for each technology type is, indeed, the more profitable package for firms within that group. Hence, the terms of the two separating contracts are necessarily distorted away from the complete information contract parameters. Alternatively, it can pool all borrowers into one group and offer a single "average" contract without attempting to extract any private information from the firms.

The properties of the contracts that result under both strategies will be studied in the remainder of this section. It will be shown that for any pooling contract, there is a set of separating contracts that is at least as profitable for firms of both types and yields strictly higher expected returns for the lender.

One might suspect that the lender could conceivably increase its total expected return by constructing more elaborate contracts intended to elicit information not only about the technology type, but also about each individual entrepreneur's reservation wage. The lender might, in fact, benefit from this knowledge. For example, it might want to identify those entrepreneurs with high reservation wages in order to charge a higher interest rate to the remainder. However, it is impossible to construct contracts that reveal this information in a one-period model.

The opportunity cost for any borrower has nothing to do with productive efficiency as a small businessman. Consequently, given that the entrepreneur has decided to enter the market, the same productive opportunities are available to all firms within the technology type. If the lender were to offer more attractive terms to entrepreneurs with higher reservation wages, all firms in the market of the same technology type would claim to have an equally high reservation wage. The lender is forestalled from gathering accurate information about any one individual's true opportunity cost of doing business.

8.3 Separating Contracts

Although the lender cannot costlessly identify each firm's true technology type, it can garner this information at a price. In order to separate the firms according to technology type, the lender must design two contracts, each of which is more profitable to only one type of firm. In other words, the lender must devise contracts that satisfy the constraint

$$(8.8) \quad E\pi_{\beta}(\hat{s}_{\beta}, \hat{r}_{\beta}) \geq E\pi_{\beta}(\hat{s}_{\alpha}, \hat{r}_{\alpha}) .$$

The lender is able to calculate the expected number of each type of firm in the market under a given set of separating contracts. Using this knowledge, it simultaneously chooses contract parameters $(\hat{s}_{\alpha}, \hat{r}_{\alpha})$ and $(\hat{s}_{\beta}, \hat{r}_{\beta})$ to maximize the sum of the total expected returns from both technology types:

$$\begin{aligned}
 (8.9) \quad E_p = \max_{j=\alpha, \beta} & \left\{ p_j(s_j) \left[\int_0^{1+r_j} \theta_j f_j(\theta_j) d\theta_j \right. \right. \\
 & + \left. \int_{1+r_j}^{\infty} (1+r_j) f_j(\theta_j) d\theta_j \right] - (1+i)Q_j + \mu_j [G_j(\gamma_j^*) - Q_j] \Big\} \\
 & + \lambda \{ p_\beta(s_\beta) \int_{1+r_\beta}^{\infty} [\theta_\beta - (1+r_\beta)] f_\beta(\theta_\beta) d\theta_\beta \\
 & - p_\beta(s_\alpha) \int_{1+r_\alpha}^{\infty} [\theta_\beta - (1+r_\alpha)] f_\beta(\theta_\beta) d\theta_\beta \Big\} ,
 \end{aligned}$$

where λ is the Lagrange multiplier associated with the type β incentive constraint.

Intuition of the standard signaling model results can be applied to this problem. In order to accomplish separation, the lender must enhance the expected profits for the type β firms under their own contract, diminish the expected profit for the firms with type β technology under the α contract, or both. The first-order conditions of equation (8.9) establish that the type β separating contract yields higher expected profit than the corresponding complete information contract for type β firms. These firms operate efficiently under both contracts, and the separating type β interest rate is less than the complete information interest rate. The terms of this contract reward the type β firms for revealing their identities. The type α contract forces firms characterized by type α technology to spend too much time in one activity and pay a higher interest rate than would obtain if the firms' private information could be credibly transmitted to the lender. Hence, type β firms benefit under the separating

scheme while the type α firms suffer a reduction in expected profits as a result of the externalities in the market.

The qualitative results of the separating interest rates can be easily established from the first-order conditions of the lender's expected return function. Rearranging the terms of the derivatives with respect to r_α and r_β results in

$$(8.10) \quad \{G_\alpha(\gamma_\alpha^*) - E\rho_\alpha(\hat{s}_\alpha, \hat{r}_\alpha)g_\alpha(\gamma_\alpha^*)\} = -\lambda \frac{p_\beta(\hat{s}_\alpha)[1 - F_\beta(1 + \hat{r}_\alpha)]}{p_\alpha(\hat{s}_\alpha)[1 - F_\alpha(1 + \hat{r}_\alpha)]}$$

and

$$(8.11) \quad \{G_\beta(\gamma_\beta^*) - E\rho_\beta(\hat{s}_\beta, \hat{r}_\beta)g_\beta(\gamma_\beta^*)\} = \lambda,$$

respectively. The LHS of both equation (8.10) and equation (8.11) were equated to zero in the complete information case. Both expressions are decreasing in the interest rate. Since the RHS of equation (8.10) is negative, it follows that $\hat{r}_\alpha > \bar{r}_\alpha$. In contrast, the fact that the LHS of equation (8.11) is positive implies that $\hat{r}_\beta < \bar{r}_\beta$. The separating scheme forces firms with type α technology to pay a higher interest rate and rewards the firms with type β technology with a lower interest rate than those determined with complete information. Thus, a portion of the lender's cost of eliciting the firms' private information has been established.

Separating contracts for type β firms specify an efficient division of each type β entrepreneur's time between business activities. This is easily seen from the first-order condition of equation

(8.9) with respect to \hat{s}_β , which simplifies to $\hat{s}_\beta = \beta$. The rationale behind this result is clear. Previously, it was shown that no firm with type α technology would prefer the complete information contract intended for the type β technology. Consequently, there is no need for the type β separating contract to serve as a deterrence mechanism for anyone in the market.

The optimal separating contracts force those firms that make relatively intensive use of the first activity to devote more than the productively efficient fraction of time to it. This result can be derived directly from the first-order conditions of the lender's expected return function with respect to s_α :

$$(8.12) \quad \frac{p'_\alpha(\hat{s}_\alpha)}{p'_\beta(\hat{s}_\alpha)} = \frac{\lambda P_\beta(\hat{r}_\alpha)}{R_\alpha(\hat{r}_\alpha)G_\alpha(\gamma_\alpha^*) + E\rho_\alpha(\hat{s}_\alpha, \hat{r}_\alpha)g_\alpha(\gamma_\alpha^*)P_\alpha(\hat{r}_\alpha)} .$$

$R_\alpha(\hat{r}_\alpha)$ is the lender's expected marginal revenue from a type α firm, conditional on output being positive, and is given by

$$(8.13) \quad R_\alpha(\hat{r}_\alpha) = E(\theta_\alpha | \theta_\alpha < 1 + \hat{r}_\alpha) F_\alpha(1 + \hat{r}_\alpha) + (1 + \hat{r}_\alpha) [1 - F_\alpha(1 + \hat{r}_\alpha)] .$$

The firm's expected profit, conditional on output being positive, is denoted $P_j(\hat{r}_j)$, and is defined as

$$(8.14) \quad P_j(\hat{r}_j) = [E(\theta_j | \theta_j > 1 + \hat{r}_j) - (1 + \hat{r}_j)] [1 - F_j(\theta_j)] , \quad j = \alpha, \beta .$$

Equation (8.10) reveals that $E\rho_\alpha(\hat{s}_\alpha, \hat{r}_\alpha)g_\alpha(\gamma_\alpha^*)$ is larger than the measure of type α firms and therefore must be positive. In turn, this

implies that the RHS of equation (8.12) is positive. The ratio on the LHS is positive whenever \hat{s}_α is smaller than β or larger than α .

The regularity condition imposed on the firms' isoprofit curves rules out the possibility of an optimal separating contract for type α firms characterized by underinvestment in the signal. The condition $\frac{\partial E\pi_\alpha}{\partial s} > \frac{\partial E\pi_\beta}{\partial s}$ for any fixed interest rate requires the isoprofit curves to assume the shapes in Figure 7(a) rather than 7(b). If the technology were represented by Figure 7(b) the lender's optimal contract offering to type α firms could be at point E, which sets $\hat{s}_\alpha < \beta$. The type β separating contract at point F is then determined by the productively efficient point on the β isoprofit curve through point E. Any separating contract that retained the same number of type α firms but instead specified $\hat{s}_\alpha > \alpha$ could decrease the lender's total expected returns due to the increased payoffs to the high-risk type β borrowers necessitated by such a move. For example, if point G were the contract offered to type α firms, the type β contract would be set at point H on a higher β isoprofit curve than contract F. However, the regularity condition is violated by the technology represented in Figure 7(b). Hence, when the technological possibilities of the two classes of borrowers are restricted to the form depicted in Figure 7(a), it can be concluded that the optimal separating contract for type α firms specifies a time share $\hat{s}_\alpha > \alpha$ as in contract A. Once again, the model leads to the standard signaling results that the lower-risk firms are forced to overinvest in the activity for which they have a relative advantage.

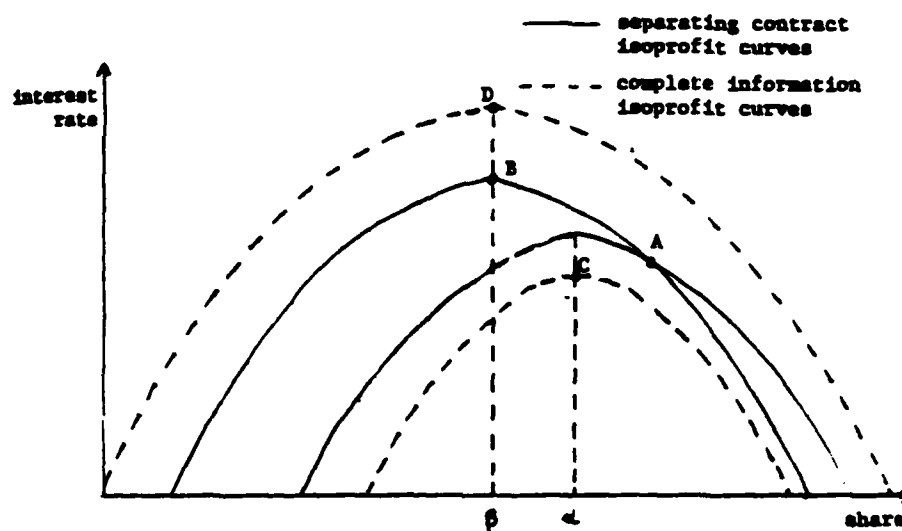


Figure 7(a): The regularity condition requires that type α firms overinvest in the signal.

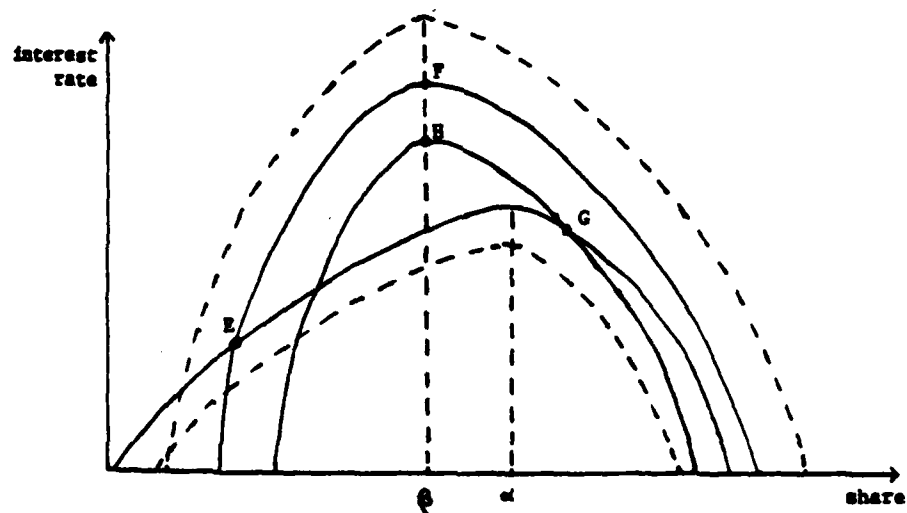


Figure 7(b): Violation of the regularity condition could lead to either overinvestment or underinvestment in the signal for type α firms.

Even though the separating strategy leads to a type α interest rate that is too high and a type β interest rate that is too low relative to the complete information contract parameters, \hat{r}_β still exceeds \hat{r}_α . This is readily determined by examining the incentive constraint which equates the expected profit for type β firms under both contracts:

$$(8.15) \quad p_\beta(\hat{s}_\beta)P_\beta(\hat{r}_\beta) = p_\beta(\hat{s}_\alpha)P_\beta(\hat{r}_\alpha) \quad .$$

It was determined that \hat{s}_β is productively efficient while \hat{s}_α forces relatively gross overinvestment in the first activity for firms with type β technology. Hence,

$$(8.16) \quad p_\beta(\hat{s}_\beta) > p_\beta(\hat{s}_\alpha) \quad .$$

From equation (8.15) it follows that

$$(8.17) \quad P_\beta(\hat{r}_\beta) < P_\beta(\hat{r}_\alpha) \quad .$$

Recalling that $P_\beta(r)$ is a strictly monotonically decreasing function, one can conclude that $\hat{r}_\beta > \hat{r}_\alpha$. The lender's retention of control over the time allocation in the business plan permits separation of the firms even though type β firms pay a higher interest rate for their borrowed funds.

The separating contract intended for firms with type α technology is less desirable than the complete information contract in both dimensions. Not only is the interest rate higher, but the firms are forced to produce inefficiently because overinvestment in one activity

is required to convince the lender of their true identity. Hence, one might expect that reverse incentive problems might arise; i.e., that the type α firms might find the type β separating contract more profitable than their own. Figure 7(a) illustrates that this is not the case. The regularity condition imposed on the firms' isoprofit curves guarantee that, despite its distortions, the type α separating contract yields higher expected profit than the type β contract for firms characterized by type α technology. Therefore, it can be concluded that the separating scheme is effective.

In a single-loan model with fixed project size, the optimal separating contracts attract relatively too few type α firms and relatively too many type β firms into the market. Both parameters of the type α contract are distorted away from the complete information contract terms. In addition to paying a higher interest rate, the type α entrepreneurs are forced to misallocate their work time. Since both of these features drive down expected firm profit, fewer type α entrepreneurs choose to become small businessmen. In contrast, the type β contract specifies efficient use of time and compensates the entrepreneurs with a lower interest rate in exchange for revealing their identities. Hence, the type β separating contract attracts a greater number of type β firms than the corresponding complete information contract.

The salient features of the lender's optimal separating contracts for a single transaction can be described as follows:

The type α firms are forced to overinvest in the activity for which they have a relative advantage and pay an interest rate above the complete information rate. Since there is no reverse incentive problem, the type β firms are allowed to produce at the productively efficient optimum. The type β borrowers are rewarded for revealing their identities with an interest rate below the complete information rate; nevertheless, the type β interest rate exceeds the rate charged to type α firms. As a result of the distortions in the contract parameters, too few type α borrowers and too many type β entrepreneurs choose to secure loans relative to the complete information case.

In a single-loan model, a pair of separating contracts can be found that dominates each possible pooling contract. This conclusion can be verified once the properties of the optimal pooling contract have been determined. Even though no rational lender would follow a pooling strategy for a single transaction, it is helpful to have characterized the properties of the pooling contract for use in the more complex multiple-transaction model.

8.4 Pooling Contracts

A pooling strategy forces all borrowers to take the same loan contract, regardless of technology type. Since the lender knows the distribution of both types of firms, it can determine the expected number of each type that will choose to borrow under any given contract. Hence, it can determine the single pair of pooling contract terms, (\hat{s}_p, \hat{r}_p) , that maximizes the sum of the total expected returns from both types of firms:

$$(8.18) \quad E\rho = \max_{j=\alpha, \beta} \left\{ p_j(s_p) \left[\int_0^{1+r_p} \theta_j f_j(\theta_j) d\theta_j + \int_{1+r_p}^{\infty} (1+r_p) f_j(\theta_j) d\theta_j \right] - (1+i)Q_j + \mu_j [G_j(\gamma_j^*) - Q_j] \right\}.$$

The first-order conditions of the lender's pooling problem reveal that inefficiencies in production almost always occur for both technology types. To compensate for the loss in returns due to the productive inefficiencies, the interest rate is driven upward. It always lies above the complete information interest rate for type α firms. In some circumstances, it even exceeds the rate that would obtain for type β firms in a complete information context.

The possible range of values that the pooling time share can assume is found from the derivative of equation (8.18) with respect to s_p :

$$(8.19) \quad \frac{p'_\alpha(\hat{s}_p)}{p'_\beta(\hat{s}_p)} = \frac{-\{R_\beta(\hat{r}_p)G_\beta(\gamma_\beta^*) + E\rho_\beta(\hat{s}_p, \hat{r}_p)g_\beta(\gamma_\beta^*)P_\beta(\hat{r}_p)\}}{\{R_\alpha(r_p)G_\alpha(\gamma_\alpha^*) + E\rho_\alpha(s_p, r_p)g_\alpha(\gamma_\alpha^*)P_\alpha(r_p)\}},$$

where $R_j(\hat{r}_p)$ and $P_j(\hat{r}_p)$ are defined as in equation (8.13) and equation (8.14), respectively.

The pooling time share is set at an intermediate value between β and α as long as $E\rho_\alpha(\hat{s}_p, \hat{r}_p)$ and $E\rho_\beta(\hat{s}_p, \hat{r}_p)$ are positive. All other terms on the RHS of equation (8.17) are unambiguously positive. Therefore, $E\rho_j(\hat{s}_p, \hat{r}_p) > 0$ ($j = \alpha, \beta$), is a sufficient condition to ensure that the entire RHS is negative. As Figure 8 illustrates, the ratio on the LHS of equation (8.17) is negative whenever \hat{s}_p is greater than the efficient time share for type β firms but smaller than the efficient division for type α firms. By assumption, $\alpha > \beta$ and

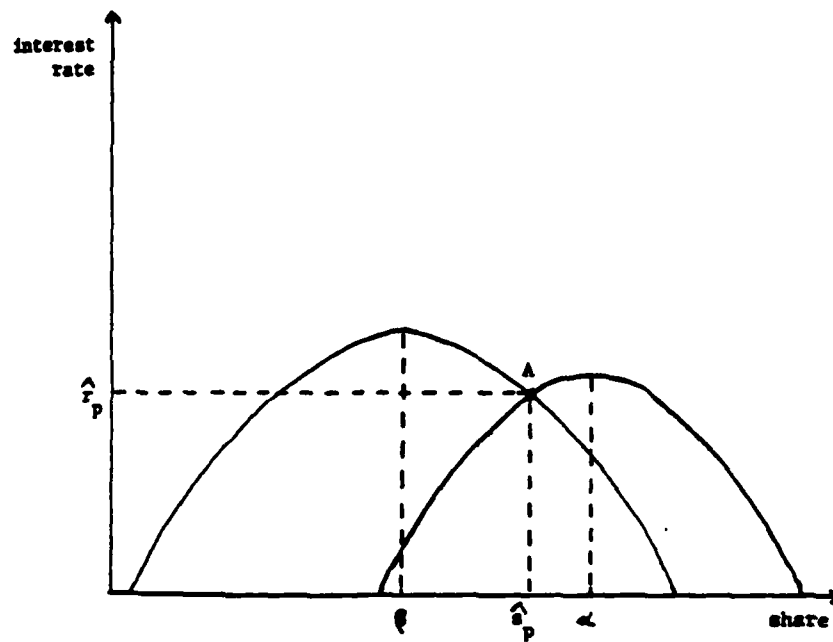


Figure 8: The time allocation for a pooling contract is set at an intermediate level when the lender's expected returns from both types of firms are positive.

$p'_\alpha(\hat{s}_p) > p'_\beta(\hat{s}_p)$. Hence, it can be concluded that $\bar{s}_\beta < \hat{s}_p < \bar{s}_\alpha$ whenever the lender's marginal return from both types of firms is positive.

Overinvestment of time devoted to one activity can occur under the terms of the optimal pooling contract if the lender suffers losses at the margin from one type of borrower. The rational lender's marginal return from at least one type of firm is positive under all circumstances. Suppose $E\rho_\alpha(\hat{s}_p, \hat{r}_p) > 0$ but $E\rho_\beta(\hat{s}_p, \hat{r}_p) < 0$. If the lender's losses from type β firms are sufficiently great, the bracketed expression in the numerator of equation (8.17) becomes negative, making the entire RHS positive. In this case, all firms are forced to overinvest in the first activity. That is, $\hat{s}_p > \alpha$. A continuity argument supports this claim. First, suppose that $E\rho_\beta(\hat{s}_p, \hat{r}_p)$ is positive. From the argument above, this implies that $\bar{s}_\beta > \hat{s}_p > \bar{s}_\alpha$. As the parameters of the type β technology are adjusted continuously so as to make the numerator of the RHS of equation (8.17) equal to zero, $\hat{s}_p = \bar{s}_\alpha = \alpha$. Once the numerator becomes negative, \hat{s}_p is increased beyond α . Hence, it is possible that the optimal pooling contract will dictate a time share set such that $\bar{s}_\beta < \bar{s}_\alpha < \hat{s}_p$.

On the other hand, all firms might be forced to overinvest in the second activity if the lender's expected return from type β firms is positive but the marginal return from a type α firm is sufficiently low that the denominator in equation (8.17) is negative. An argument analogous to the one in the preceding paragraph leads to the conclusion that, in such a case, the time share of the optimal pooling contract is set such that $\hat{s}_p < \bar{s}_\beta < \bar{s}_\alpha$.

Overinvestment of time in one of the activities occurs only when the lender suffers relatively large expected losses from the firms with type j technology. By specifying such a contract, the lender makes the productive efficiency so low for these firms that relatively few type j entrepreneurs choose to borrow. Although the firms of the other technology type are not producing at their most efficient point under this sort of pooling scheme, the reduction in their expected profit is less severe; consequently, the lender relinquishes less of their business.

The optimal pooling interest rate always lies above the complete information interest rate for type α firms. Under some circumstances, the lender charges a pooling interest rate that even exceeds \bar{r}_β . This can be determined by studying the derivative of equation (8.16) with respect to r_p :

$$(8.20) \quad p_\alpha(\hat{s}_p)[1 - F_\alpha(1 + \hat{r}_p)]\{G_\alpha(\gamma_\alpha^*) - E\rho_\alpha(\hat{s}_p, \hat{r}_p)g_\alpha(\gamma_\alpha^*)\} \\ = -p_\beta(\hat{s}_p)[1 - F_\beta(1 + \hat{r}_p)]\{G_\beta(\gamma_\beta^*) - E\rho_\beta(\hat{s}_p, \hat{r}_p)g_\beta(\gamma_\beta^*)\}.$$

To simplify the analysis, assume that $g_j(\gamma_j)$ is constant and $G_j(\gamma_j^*)$ is linear in expected profit. That is,

$$(8.21) \quad G_j(\gamma_j^*) = g_j E\pi_j(\hat{s}_p, \hat{r}_p), \quad j = \alpha, \beta.$$

Then equation (8.20) can be restated as

$$(8.22) \quad p_\alpha(\hat{s}_p)[1 - F_\alpha(1 + \hat{r}_p)]g_\alpha\{E\pi_\alpha(\hat{s}_p, \hat{r}_p) - E\rho_\alpha(\hat{s}_p, \hat{r}_p)\} \\ = -p_\beta(\hat{s}_p)[1 - F_\beta(1 + \hat{r}_p)]g_\beta\{E\pi_\beta(\hat{s}_p, \hat{r}_p) - E\rho_\beta(\hat{s}_p, \hat{r}_p)\}.$$

Note that under the terms of the complete information contracts, the lender's marginal return is equated to expected firm profit for each of the respective technology types. When the lender chooses to pool, however, condition (8.22) requires that either

$$(8.23) \quad E\pi_{\alpha}(\hat{s}_p, \hat{r}_p) < Ep_{\alpha}(\hat{s}_p, \hat{r}_p) \quad \text{and} \quad E\pi_{\beta}(\hat{s}_p, \hat{r}_p) > Ep_{\beta}(\hat{s}_p, \hat{r}_p)$$

or

$$(8.24) \quad E\pi_{\alpha}(\hat{s}_p, \hat{r}_p) > Ep_{\alpha}(\hat{s}_p, \hat{r}_p) \quad \text{and} \quad E\pi_{\beta}(\hat{s}_p, \hat{r}_p) < Ep_{\beta}(\hat{s}_p, \hat{r}_p) .$$

Each of these two cases will be considered in turn.

First, suppose that the conditions in (8.23) hold. At the pooling time share, the lender is dealing with too few type α firms and too many type β firms. Since each firm's expected profit is a multiple of the probability of positive outcome but the lender's cost is fixed, independent of this probability, inefficiencies in production have a greater impact on the lender's return than on the firm's profit. Thus, moving \hat{s}_p away from α without altering the complete information interest rate decreases the lender's marginal return proportionately more than the type α firm's expected profit. That is,

$$(8.25) \quad E\pi_{\alpha}(\hat{s}_p, \bar{r}_{\alpha}) > Ep_{\alpha}(\hat{s}_p, \bar{r}_{\alpha}) .$$

Recall that each firm's expected profit function is monotonically decreasing in the interest rate, but the lender's expected return is monotonically increasing in the same argument. Therefore, to meet the

conditions of this case, it must be true that the interest rate is raised above \bar{r}_α . That is $\hat{r}_p > \bar{r}_\alpha$.

Causing inefficiencies in the type β production process also reduces the lender's expected marginal return more than the expected profit for type β firms. This leads to the condition

$$(8.26) \quad E\pi_\beta(\hat{s}_p, \bar{r}_\beta) > E\rho_\beta(\hat{s}_p, \bar{r}_\beta) .$$

Since the inequality holds in the required direction, the size of \hat{r}_p relative to \bar{r}_β is indeterminate. Therefore, in the first case it can be concluded that $\hat{r}_p > \bar{r}_\alpha$ but $\hat{r}_p \geq \bar{r}_\beta$.

Suppose instead that the lender offers a pooling contract that induces too many type α firms but too few type β firms to apply for loans. That is, the conditions stated in (8.24) are in effect. The inefficiencies in production lead to the inequality

$$(8.27) \quad E\pi_\alpha(\hat{s}_p, \bar{r}_\alpha) > E\rho_\alpha(\hat{s}_p, \bar{r}_\alpha) ,$$

which imposes no definitive relationship between \hat{r}_p and \hat{r}_α . However, forcing the type β firms to produce inefficiently results in:

$$(8.28) \quad E\pi_\beta(\hat{s}_p, \bar{r}_\beta) > E\rho_\beta(\hat{s}_p, \bar{r}_\beta) .$$

In order to meet the conditions defining this case, the interest rate must be forced above \bar{r}_β . Therefore, when the pooling contract attracts relatively too many type α firms and too few type β firms, $\hat{r}_p > \bar{r}_\beta$. That is, the optimal pooling interest rate exceeds both of the complete information interest rates.

The lender suffers reductions in expected returns due to the productive inefficiencies inherent under the terms of a pooling contract. To compensate for some of these losses, the interest rate is increased. The higher rate increases the expected returns from those firms that do borrow. However, it deters some potential entrepreneurs from securing loans. Because the trade-off between these two considerations is indeterminate, only weak results about the range of the optimal pooling interest rate can be established.

It has been determined that overinvestment in the instrument used as a signal is possible even under the terms of the optimal pooling contract. The incentive constraint imposed on separating contracts is not the only mechanism that produces this phenomenon. The lender might find that overinvestment is the least costly way of compensating for its inability to acquire private information held by each entrepreneur. If it could classify borrowers, it would refuse to lend funds to the undesirable firms at the pooling contract terms. Since this is impossible, the optimal pooling contract is set in such a way that proportionately few of these entrepreneurs choose to borrow.

Since inefficiencies are almost always imposed on the productive processes of both technology types under the terms of the optimal pooling contract, the risk of default increases for each borrower relative to the complete information case. In return, the lender requires some compensation in the form of a higher interest rate for at least some firms. This, however, diminishes the total volume of loan demand. The optimal trade-off between the increase in marginal return but decrease

in total number of borrowers leads to only ambiguous comparative statements about contract parameters. Under the assumption that the measure of firms is linear in expected profit, the pooling interest rate always exceeds the complete information interest rate for the type α technology. It might, but need not always, exceed the rate charged to type β firms in a complete information context.

In summary, the optimal pooling contract in a single-loan model has the following features:

The pooling time share in a single-loan model is set at an intermediate value between α and β if the lender's expected return from each technology type is positive. If the lender anticipates sufficiently large losses from one group of borrowers, the time share is set at a level that drives most of the undesirable firms out of the market. Since productive inefficiencies are inherent in the pooling contract, the lender must be compensated for its increased risk. Therefore, the pooling interest rate always exceeds the complete information interest rate for type α firms. It may, but need not always, exceed the complete information type β interest rate.

Now that the properties of both the separating and pooling contracts have been established for a single-loan model, it will be shown that the separating strategy always dominates pooling as long as multiple transactions are prohibited.

8.5 Separating Contracts Dominate Pooling in a Single-Loan Model

The separating strategy always dominates pooling when only one transaction between the lender and each borrower is possible. Given any optimal pooling contract, there is always a pair of separating contracts that would yield at least as high an expected profit for firms of both

technology types and a higher expected return for the lender. To see this, consider Figure 9. Suppose the optimal pooling contract is determined to be point A. Instead of extending only this contract, the lender could offer both the contract at point A and the one at point B. Type β firms would be indifferent between the two, but type α firms would remain at point A. Hence, firms of both technology types would earn the same expected profit as they would under the terms of the single pooling contract. The lender, of course, would earn the same expected return from the type α firms under the new scheme. Moreover, its expected return from the type β firms would be augmented due to the increase in efficiency of production and higher interest rate at point B. The new set of contracts might not be the optimal separating contracts, but they are more profitable for the lender than the single pooling contract. A similar line of argument holds for any pooling contract as long as the optimal pooling time share requires inefficient production techniques for one of the technology types. Thus, it can be concluded that a rational lender always chooses to separate borrowers when only one transaction is relevant.

Briefly, this result can be stated as follows:

For every pooling contract that might be offered in a single-loan model, a pair of separating contracts can be constructed which yields higher expected returns for the lender. The domination of separating over pooling contracts can be attributed to the productive inefficiencies imposed on all firms under the terms of the pooling contract.

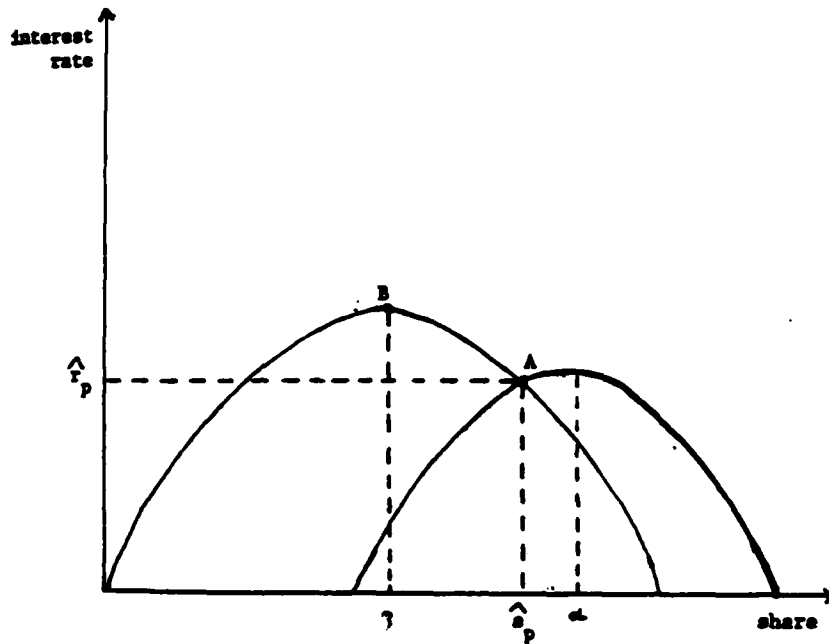


Figure 9: Pooling contracts are dominated by separating contracts for a single transaction in the fixed-loan-size model.

9. Perfect Strategies in a Two-Period Model with Loan Size Fixed

The potential for the establishment of standing relationships and multiple transactions between the lender and a given borrower opens new strategic possibilities for both parties. Signaling remains an important component of the negotiations because the borrower might still have some private information at each stage. The significance of the role of the signal in comparison to a single-transaction model, however, is not immediately obvious. Through observation of early outcomes, the lender naturally learns some facts about its clients with which it can form more accurate assessments of entrepreneurial skill. This would seem to mitigate the need to force artificial means of transmitting information, such as overinvestment of time in some activity. However, early acquisition of information relevant to firm technology increases in value when the lender knows there is an opportunity to make use of the knowledge in negotiating subsequent loan contracts. All of these considerations raise new incentive issues in the repeated context.

In the model that follows, every firm potentially can secure a loan in each of two successive periods. The entrepreneur applies for a loan in the first period only if the expected profit over the two-period horizon exceeds his accumulated reservation wage. Once the entry decision has been made, a second loan is secured only if the expected profit under the new contract terms exceeds the reservation wage for one period. The contract terms available for the second loan might be restricted by the outcome of the first-period project. No firm is

eligible for the second loan without first going through the initial round of negotiations.

A crucial determinant of the lender's expected return in each period is the relative number of type α and type β firms that actually borrow at each stage. A borrower might be able to take advantage of the lender's ignorance at the outset. For example, it might obtain a loan in the first period, knowing that none of the terms offered in the second period will provide great enough expected profit to induce the firm to borrow again. Alternatively, some firms might borrow in the first period even when first-period expected profit is lower than the opportunity cost of doing business because acceptance of the first loan contract is a necessary prerequisite to a more lucrative contract in the second round. Such considerations drive the lender's strategic choices of loan contract terms as well as each borrower's decisions.

First- and second-period contracts specify both the interest rate and the division of the entrepreneur's time between two business activities. The information available to the lender in setting these terms is the same in the first period as it was at the outset of the single-loan model. Before the start of the second period, the lender receives an accurate report of the quality of each borrower's first-stage output. Although the lender's information set is improved, the identification problem is not solved due to the randomness of the quality variable. These output data can be used to devise the contract terms offered in the second period. Any borrower's choice of contracts can be

restricted, contingent on its first-stage profitability. Both parties take this into account in formulating their initial decisions.

Both perfect and imperfect strategies will be investigated. Perfect strategies isolate one of the advantages of standing relationships between borrowers and creditors; namely, the value of permitting low-risk borrowers the opportunity to distinguish themselves early in the negotiations. Imperfect strategies add the dimension of the value to the lender of gaining experience with several generations of borrowers and building a reputation for enforcing incentive schemes. Imperfection broadens the set of strategic choices open to the lender and significantly alters the form of the equilibrium contracts.

Perfect strategies, discussed in this chapter, restrict the lender to self-enforcing threats and promises about the second loan contracts. Self-enforcing commitments are ones which would be optimal for the lender to implement on the basis of the expected return from the second loan only, where the expectation incorporates the information gained from the first project outcome. While the information learned by the lender from the first project return can never be harmful, it will be shown that under the stringent restriction of perfect strategies, standing relationships between the lender and borrowers actually diminishes the expected returns of the lender under some specifications of the exogenous technological parameters. The cost of separating borrowers at the beginning of the two-loan horizon can be great enough to make a first-period pooling contract viable.

Imperfect strategies, those in which unrestricted threats and promises can be used by the lender, will be studied in Section 10. Here the lender can commit itself to contract terms even if there is some chance that these terms will prove to be less than profit maximizing once the project returns from the first loans are known. In particular the lender potentially can devise contracts that take advantage of the differences in reservation wages within each technological category. Imperfection clearly cannot decrease the value of a standing relationship for the lender because it simply increases the set of feasible contract choices. Perfect strategies are still permitted; the option of ignoring the consequences of repetition is also open. In fact, with imperfect strategies, the lender is able to strictly improve its position by the establishment of long-term relationships with borrowing firms.

The equilibrium contracts that obtain when perfection is imposed on the lender's strategies will be characterized in this chapter. Perfection requires that the lender maximize its remaining expected profit at each stage of contracting, given all the information accumulated to that point. No other course of action is credible. Regardless of any claims that the lender might make to the contrary, all borrowers base their strategic decisions on the assumption of perfect lender behavior.

Section 8 established that separating strategies always dominate pooling when each borrower can secure only one loan. If repetition is permitted, the same conclusion depends on the restrictions placed on the

lender's decisions. Perfection proves to be such a strong requirement in this model that a pooling strategy yields a higher expected return for the lender than separating contracts under some specifications of the exogenous variables when multiple transactions are anticipated.

Many of the features of the optimal single-loan contracts remain relevant for the equilibrium contracts in the repeated transaction model. When the lender separates firms at the outset, the type α firms must suffer productive inefficiencies and higher interest payments while the type β firms receive payoffs in return for revealing their identities. The optimal pooling contract at the outset of a two-loan horizon could specify a time share at an intermediate level between the complete information shares for the two technology types. However, if the lender suffers large losses from one group of firms, overinvestment could be required of all borrowers. Although the qualitative properties of the two-period contracts are very similar to those determined for the single-loan case, the intertemporal trade-offs between expected return in the first period and improved information in the second, in addition to the intertemporal dependencies imposed on the contract parameters, make the results more complicated to ascertain.

The lender is precluded from segmenting the entrepreneurs on the basis of their reservation wages when perfection is imposed. The reservation wage does not influence potential firm profitability, given that the entrepreneur has chosen to act as a small businessman. The lender would have to purchase information about the opportunity cost by offering more attractive contract terms to some firms. Any such attempt that

involved contracts extending over two periods would fail to separate firms because all the firms would choose the more profitable package, regardless of the level of the reservation wage. Any attempt to elicit information that included a profitable contract in the first period and no loan in the second would not be credible and, therefore, would be ruled out by perfection.

9.1 Separating Contracts for the First Loan

The optimal contracts that induce separation of technological types of borrowers at the outset of the two-period horizon are analyzed in this subsection. Assuming that separation is accomplished in the first period, every borrower's technological type has been revealed before the second-period contract terms are extended. The restriction of perfection then forces the lender to incorporate this information into the second-period contract offerings.

An overview of the reasoning used in selecting the lender's contract terms and a flavor of the properties of the optimal separating contracts will be presented before characterizing the solution formally.^{19/} The lender employs dynamic programming, beginning with the last round of transactions, to maximize its expected returns over the two-period horizon. The second-period parameters are derived as functions of the number of first-period borrowers. Since each borrower's technological type has been unambiguously identified by its choice of first-period contract terms, no firm is denied funds in the second period. It will be shown that all type α borrowers remain in the market for both periods, but some type β entrepreneurs exit after the

initial round of transactions. Once the lender's decision rules have been determined for the second period, the first-period contract terms are chosen, taking into account their influence on second-period returns.

Perfection eliminates nearly all flexibility in the choice of second-stage contract parameters when the lender chooses to separate at the outset. It constrains the lender to use all of its information to maximize its expected return from each point in time forward. Since identification is achieved through separation at the outset, knowledge of the first-period project realization is irrelevant. Within technological categories, all firms are treated identically regardless of first-period profitability. Only complete information contracts are credible in the second stage. Furthermore, the lender can do nothing to attract more than the original set of entrepreneurs into the market. Its only remaining decision at that point is whether to retain all of the firms of a particular technological type or drive a fraction of them out of business.

Given all of the restrictions on the second-stage decisions, the lender must build nearly all of the separating incentives into the first-period contracts. The basic qualitative properties of the separating contracts derived in the one-loan model hold for the first-round loans offered in the repeated context. Namely, the type α contracts are less profitable for the firms with type α technology; and the type β contracts are more profitable for the type β firms than the corresponding complete information contracts would be. Hence, from the

lender's viewpoint, there are too few type α firms and too many type β firms in the market.

In order to determine the binding incentive constraints needed to effect separation, the lender must consider both the actions of type β firms that plan to borrow only for the first period and the decisions of those who plan to borrow twice. It will be shown that if the constraint relevant for the latter group is met, the constraint induced by the former group is automatically satisfied.

Perfection requires the lender to offer second-period contracts that are as similar to single-loan complete information contracts as possible, subject only to the number of each type of entrepreneur who chose to borrow initially. Since too few type α entrepreneurs secure loans in the first period, it follows that none of those borrowers will be eliminated by the terms of the second-period separating contracts. Maximizing the lender's second-period return requires minimizing the cost of retaining $Q_{\alpha 1}$ type α firms.^{20/} That is, the lender sets the terms of the second-period type α contract such that

$$(9.1) \quad E\pi_{\alpha}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) = E\pi_{\alpha}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2})$$

Since perfection requires that $\bar{s}_{\alpha 2}$ be set at its productively efficient level, it must follow from equation (9.1) that the second-period type α interest rate exceeds the first-period rate.

The type β contract offered in the second period is exactly the same as the single-loan complete information contract since the lender is not constrained by too few first-period type β borrowers. Some

type β entrepreneurs are induced to secure the first loan because of the reward for revelation of technological information, even though they have no intention of applying for the second loan. The reward required to accomplish separation in the first period eliminates binding quantity constraints for type β borrowers in the second round of transactions.

Perfection and resolution of the incentive constraints nearly determine the lender's problem for a two-period separating strategy. Maximization of the lender's total expected return with respect to all contract terms confirms the intuition provided above. This is most easily accomplished in two stages, beginning with the second period. Given first-period decisions, the lender's second-period problem is constrained by perfection and the number of first-period borrowers within each technological category. No incentive constraint need be considered since all inducements to reveal technological identity are incorporated into the first-period contract terms. Hence, the lender's second-period expected return function can be written

$$\begin{aligned}
 (9.2) \quad E\rho^2 = \max_{j=\alpha, \beta} \{ & (p_j(s_{j2})) \left[\int_0^{1+r_{j2}} \theta_{j2} f_j(\theta_{j2}) d\theta_{j2} \right. \\
 & + \left. \int_{1+r_{j2}}^{\infty} (1+r_{j2}) f_j(\theta_{j2}) d\theta_{j2} \right] - (1+i) \} Q_{j2} \\
 & + \mu_{j2} [G_j(\gamma_{j2}^*) - Q_{j2}] + \tilde{\mu}_{j2} [Q_{j1} - Q_{j2}] \}.
 \end{aligned}$$

The number of firms in the market in the second period is constrained in two ways. Not only must the lender make the contract terms

profitable enough to retain Q_{j2} firms, but no more than the number of first-period borrowers of each technology type can be attracted. The latter constraint binds only from above. If, from the lender's viewpoint, too many firms of technology j borrowed in the first period, Q_{j1} exceeds Q_{j2} . Complementary slackness then requires $\tilde{\mu}_{j2} = 0$. Hence, the preceding argument determines that $\tilde{\mu}_{\alpha 2}$ is positive while $\tilde{\mu}_{\beta 2}$ is zero.

The first-order conditions of equation (9.2) reveal the expected results. Firms of both technology types produce efficiently in the second period. That is, $\bar{s}_{\alpha 2} = \alpha$ and $\bar{s}_{\beta 2} = \beta$. The interest rate for type β firms is set at the same level as in a one-period complete information problem. However, the reduced number of first-period type α borrowers forces the second-period type α interest rate to a higher level than in a comparable single-transaction problem.

Conclusions about the respective second-period interest rates follow from some simple substitution, yielding the condition

$$(9.3) \quad \frac{G_j(\gamma_{j2}^*)}{g_j(\gamma_{j2}^*)} - E p_j(\bar{s}_{j2}, \bar{r}_{j2}) = -\tilde{\mu}_{j2}, \quad j = \alpha, \beta.$$

Recall that $\tilde{\mu}_{\beta 2} = 0$. Hence, equation (9.3) is identical to equation (8.7) in the single-transaction model for technology β . Since both type β contracts are productively efficient, the interest rates must be equal.

The same result is not true for type α firms since both quantity constraints bind for them ($\mu_{\alpha 2} > 0$ and $\tilde{\mu}_{\alpha 2} > 0$). This implies that

the second-period type α interest rate must be higher than the single-loan complete information interest rate. To see this, note that the time share, \bar{s}_α , is efficient both in the one-loan and in the second period of the two-loan problem. Equation (8.7) from the single-loan model sets marginal lender return equal to the ratio $\frac{G_\alpha(\gamma_\alpha^*)}{g_\alpha(\gamma_\alpha^*)}$. Here their difference is equated to a negative constant. Both terms on the LHS of equation (9.3) are decreasing in the interest rate; therefore, $\bar{r}_{\alpha 2}$ must be higher than the comparable interest rate in the one-loan problem.

Before selecting the parameters of the first-period separating loans, the lender must determine the incentive constraints that need to be met in order to insure the cooperation of all borrowers. Because of the differences in the opportunity costs among type β entrepreneurs, the motivating incentives are not uniform for all first-period type β borrowers. Those firms that plan to secure a loan only in the initial period are willing to reveal their technological identities if the first-period contract terms satisfy the inequality

$$(9.4) \quad E\pi_\beta(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) > E\pi_\beta(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) .$$

If this constraint is met exactly, some other type β firms that were infra-marginal with respect to their first-period borrowing decision might still be tempted to cheat. Although they are indifferent between the α and β contracts in the first period, their expected profit over the two periods is higher under the two type α contracts than under the terms of the two type β contracts. Figure 10 is useful

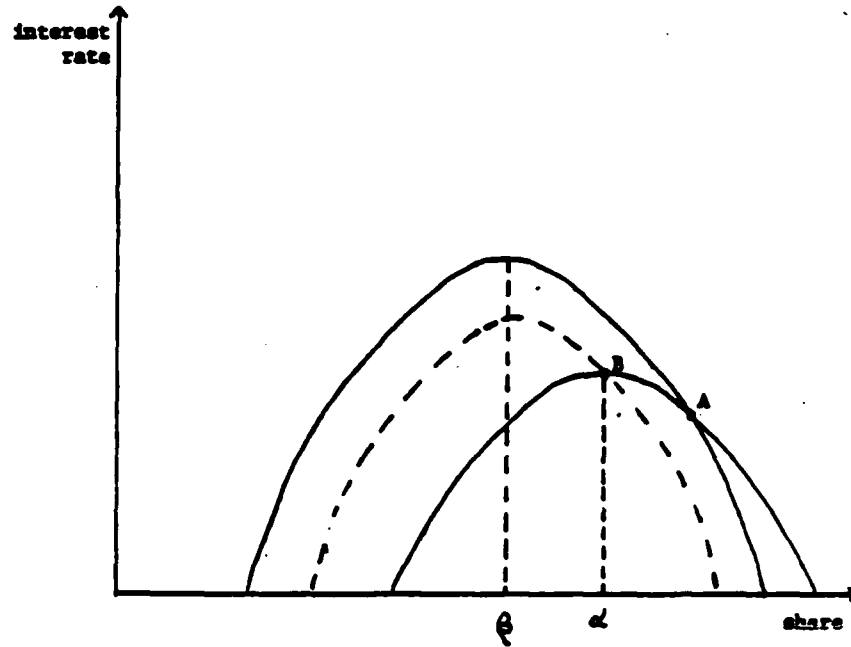


Figure 10: The binding incentive constraint is determined by those type θ firms that plan to borrow twice under perfect separating strategies.

for illustrating this argument. Point B, which represents the second-period type α contract, is on a higher isoprofit curve than point A, the first-period type α contract, for firms characterized by type β technology. Therefore, satisfaction of inequality (9.4) does not guarantee complete separation of technological types when more than one period of contracting is anticipated.

The structure imposed by perfection aids in determining the binding incentive constraint. From equation (9.1), both of the type α contracts must be on the same isoprofit curve for firms with type α technology. Furthermore, the first-period contract specifies overinvestment in order to attain separation while the second-period contract is productively efficient due to perfection. Hence, as is readily seen in Figure 10, expected profit for type β firms is greater under the terms of the second-stage type α contract than under the first-stage one. Not only must the lender meet the constraint in inequality (9.4), but it must also meet the stronger condition that

$$(9.5) \quad E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) + E\pi_{\beta}(\bar{s}_{\beta 2}, \bar{r}_{\beta 2}) \geq E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + E\pi_{\beta}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2}) .$$

Rewriting this constraint as

$$(9.6) \quad E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) - E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) \geq E\pi_{\beta}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2}) - E\pi_{\beta}(\bar{s}_{\beta 2}, \bar{r}_{\beta 2})$$

it is easy to see that satisfaction of inequality (9.4) is insufficient to accomplish separation for all type β firms in the market since the RHS of (9.6) must be greater than zero. Expected profit for type β technology must be greater under the terms of the more desirable of the

two type α contracts than under the specifications of the less desirable of the two type β contracts. However, weak inequality in (9.5) assures strict inequality in (9.4).

Having ascertained the necessary incentive constraints for type β borrowers and the second-period contract terms as functions of first-period decisions, the lender can select the first-period contract terms. The choice of these parameters is complicated by intertemporal considerations. The second-period type α interest rate depends on the number of type α firms that borrow initially. Hence, it is a function of both parameters of the first-period type α contract. The derivatives of this function can be determined implicitly from condition (9.1). Virtually all of the inducement to separate must take place in the first-period, even though both type α contracts affect each type β firm's incentive to cheat. Consequently, the lender must carefully choose the first-period contract terms, taking into account the functional relationships between first-and second-period parameters. The first-period problem can be stated as:

$$\begin{aligned}
 (9.7) \quad E\rho = & \max_{j=\alpha, \beta} \sum \{ p_j(s_{j1}) \left[\int_0^{1+r_{j1}} \theta_{j1} f_j(\theta_{j1}) d\theta_{j1} \right. \\
 & + \int_{1+r_{j1}}^{\infty} (1+r_{j1}) f_j(\theta_{j1}) d\theta_{j1} \left. \right] - (1+i) Q_{j1} \\
 & + \mu_{j1} [G_j(\gamma_{j1}^*) - Q_{j1}] \} + E\rho^2 \\
 & + \lambda \sum_{t=1,2} \{ p_{\beta}(s_{\beta t}) \int_{1+r_{\beta t}}^{\infty} [\theta_{\beta t} - (1+r_{\beta t})] f_{\beta}(\theta_{\beta t}) d\theta_{\beta t} \\
 & - p_{\beta}(s_{\alpha t}) \int_{1+r_{\alpha t}}^{\infty} [\theta_{\beta t} - (1+r_{\alpha t})] f_{\beta}(\theta_{\beta t}) d\theta_{\beta t} \} .
 \end{aligned}$$

Just as in the one-loan separating problem, both parameters of the first-stage type α contract are distorted away from the complete information contract terms. However, the determination of the conditions imposed on $\hat{s}_{\alpha 1}$ and $\hat{r}_{\alpha 1}$ are complicated by their influence on the second-period type α interest rate. Substituting the first-order conditions from the second-period contracts into the derivatives with respect to $s_{\alpha 1}$ and $r_{\alpha 1}$ yields, in turn,

$$\begin{aligned}
 (9.8) \quad & \frac{p'_{\alpha}(\hat{s}_{\alpha 1})}{p'_{\beta}(\hat{s}_{\beta 1})} \\
 & = \frac{\lambda P_{\beta}(\hat{r}_{\alpha 1})}{R_{\alpha}(\hat{r}_{\alpha 1}) Q_{\alpha 1} + [\mu_{\alpha 1} g_{\alpha}(\gamma_{\alpha 1}^*) - \lambda \frac{p_{\beta}(\bar{s}_{\alpha 2}) [1 - F_{\beta}(1 + \bar{r}_{\alpha 2})]}{p_{\alpha}(\bar{s}_{\alpha 2}) [1 - F_{\alpha}(1 + \bar{r}_{\alpha 2})]}] P_{\alpha}(\hat{r}_{\alpha 1})}
 \end{aligned}$$

and

$$\begin{aligned}
 (9.9) \quad Q_{\alpha 1} - [\mu_{\alpha 1} g_{\alpha}(\gamma_{\alpha 1}^*) - \lambda \frac{p_{\beta}(\bar{s}_{\alpha 2}) [1 - F_{\beta}(1 + \bar{r}_{\alpha 2})]}{p_{\alpha}(\bar{s}_{\alpha 2}) [1 - F_{\alpha}(1 + \bar{r}_{\alpha 2})]}] \\
 = - \lambda \frac{p_{\beta}(\hat{s}_{\alpha 1}) [1 - F_{\beta}(1 + \hat{r}_{\alpha 1})]}{p_{\alpha}(\hat{s}_{\alpha 1}) [1 - F_{\alpha}(1 + \hat{r}_{\alpha 1})]} .
 \end{aligned}$$

Type α firms must overinvest in one activity under the terms of the first-period contract. To see this, note that the term in square brackets, which appears in both conditions (9.8) and (9.9), must be greater than the measure of type α firms in the market and, therefore, must be positive. From this it follows that both the numerator and denominator of the RHS of equation (9.8) are positive. The LHS is positive when $\hat{s}_{\alpha 1}$ lies outside the interval $[\beta, \alpha]$. Therefore, using the same argument as in the one-period problem, one can conclude that $\hat{s}_{\alpha 1} > \alpha$.

The magnitude of the distortions in the first-period time allocation, however, may be larger or smaller than the distortions in a comparable single-period contract because two aspects of the intertemporal complications work in conflicting directions. Improvements in the efficiency of the first-period time allocation induce reductions in the second-period interest rate. Both of these adjustments intensify the incentive for type β firms to cheat, and therefore increase the value of λ . Hence, the distortions in $\hat{s}_{\alpha 1}$ tend to be compounded. However, the desire to retain a sufficient number of type α entrepreneurs in the market tends to pull the first-period time share back toward the productively efficient allocation. Neither of these effects unambiguously dominates the other, so the productive inefficiency built into

$\hat{s}_{\alpha 1}$ could exceed or fall short of the inefficiency required in a single-transaction model.

The indirect effect of the first-period time share through the second-period interest rate is accounted for by the relative probability of positive firm profit from each technology type under the terms of the second-period type α contract, multiplied by the marginal cost of cheating,

$$(9.10) \quad \lambda \frac{p_{\beta}(\bar{s}_{\alpha 2}) [1 - F_{\beta}(1 + \bar{r}_{\alpha 2})]}{p_{\alpha}(\bar{s}_{\alpha 2}) [1 - F_{\alpha}(1 + \bar{r}_{\alpha 2})]} .$$

A decrease in the first-period time distortion leads to a lower second-period type α interest rate because the total number of first-period type α borrowers moves closer to the complete information optimal level. Given that the type β technology is inherently more risky, the decrease in $\bar{r}_{\alpha 2}$ would tend to increase the ratio of probabilities in expression (9.10). Moreover, the increased desirability of both type α contracts exacerbates the incentive for type β firms to cheat, raising the level of the Lagrange multiplier. Both of these effects increase the RHS of equation (9.8), implying further distortions in the first-period time share. Hence, movements toward productive efficiency in the first period are held in check by the incentive constraint even more forcefully in the multi-period problem than in the single-loan transaction.

The reduction in the first-period expected returns caused by too few first-period type α borrowers works counter to the incentive

considerations. An increase in $\mu_{\alpha 1}$ increases the denominator on the RHS of equation (9.8). Hence, as the quantity constraint becomes more tightly binding, $\hat{s}_{\alpha 1}$ is set closer to the productively efficient allocation, α . The first-period quantity constraint takes on added importance in the repeated context because it constrains expected returns in subsequent periods. Consequently, the lender attempts to attract more type α firms into the market with higher productive efficiency at the outset if the expense from forfeiting their business over the entire two-period horizon is substantial.

The average of the interest rates charged to type α firms over the two-period horizon exceeds the complete information rate when the lender chooses to separate at the outset. However, it need not be true that the first-period rate exceed \bar{r}_{α} . These conclusions can be deduced by further substitution of first-order conditions into equation (9.9) to arrive at

$$(9.11) \quad \frac{G_{\alpha}(\gamma_{\alpha 1}^*)}{g_{\alpha}(\gamma_{\alpha 1}^*)} < \frac{1}{2} [E\rho_{\alpha}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + E\rho_{\alpha}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2})] .$$

In a complete information setting, the two expressions set out above would be equated. Since the LHS of the inequality decreases in the interest rate while the RHS increases, it must be true that the average of the type α interest rates over the two periods exceeds the complete information rate. Earlier it was established that $\bar{r}_{\alpha 2}$ must exceed $\hat{r}_{\alpha 1}$. Hence, it is possible to have $\hat{r}_{\alpha 1} < \bar{r}_{\alpha} < \bar{r}_{\alpha 2}$.

The rationale for these results is clear. The lender compensates for the cost imposed by incomplete information by charging interest rates that, on average, exceed the complete information rate. The cost itself is represented by the inefficiency in the type α time specification required to prevent type β firms from selecting the wrong contract. In order to avoid driving too many of the type α firms from the market because of the productive inefficiencies in the first period, and thereby diminishing the number of profitable second-period type α contracts, the first-period type α interest rate is set at a relatively low level. In fact, it might even fall below the complete information rate. The lender must endure decreased profits in the first period before deriving any benefit from the lucrative second-period loans.

The first-order conditions for the first-period type β contract parameters are functionally identical with those of the one-period separating problem. The type β firms produce efficiently; i.e., $\hat{s}_{\beta 1} = \beta$. The incentive constraint, however, is more difficult to meet in this case; so the interest rate must be lower than in a comparable one-period negotiation. The type β firms know that they are forfeiting an even more enticing type α contract in the second period. Thus, the reward for revelation of identity must be correspondingly higher. Consequently, the lender draws an even greater number of unwanted type β firms into the market for the first period.

When the project size is fixed exogenously, the results of the lender's perfect separating strategy are similar to those of the

variable-loan size model. The salient features of the equilibrium contracts can be summarized as follows:

The lender's optimal strategy when separating borrowers at the outset of a two-loan problem calls for type α contracts that require overinvestment in one activity in the first period and too high an interest rate, averaged over the two periods, relative to the complete information optimum. Although most of the distortions induced by the incentive constraint appear in the first-period contracts, a portion of the restraining effect comes through the higher second-period type α interest rate. Since no new entrepreneurs can be attracted in the second period, perfection requires that the interest rate be set at the level that makes the first-period marginal type α borrower just indifferent between operating as a small businessman and leaving the market in the second period as well. The higher interest rate makes the second-period type α contract less attractive to both technology types. Hence, it aids the lender in satisfying the incentive constraint.

All of the reward to the type β firms for revealing their true technological identity, and thereby forfeiting type α contracts for two periods, must come through the first-period type β contract. The payoffs to the type β firms are costly to the lender in two respects. It receives a lower interest rate from the first loan it extends to those firms that would borrow even at the complete information contract terms. In addition, the attractive terms of the first-period contract draw additional type β firms into the market. These firms have no intention of applying for a second loan. They merely enter to exploit the lender's disadvantage in identifying a firm's technological type a priori.

Perfection imposes a high cost on the lender for separating borrowers at the outset of repeated contracting. In fact, the cost might be high enough to make a pooling strategy optimal in the first period despite its inherent productive inefficiencies. Such a contract involves no effort to elicit private information from the entrepreneurs; all borrowers are treated identically in the first period. In the next

subsection, the properties of this strategy will be investigated and an attempt will be made to determine conditions under which pooling is likely to be the optimal choice for first-period contracts.

9.2 Pooling Contracts for the First Loan

All first-term borrowers are treated identically under the terms of a pooling contract. No rewards need be paid since no private information is sought. Therefore, even though productive inefficiencies for all borrowers are inherent when a pooling strategy is followed, the lender still might find that pooling contracts in the first stage lead to higher expected returns over the two-period horizon when perfection is required.

The properties of the optimal pooling contract when the lender is restricted to perfect strategies are studied in this subsection. The features of the one-period pooling contract are retained by the first-period contract offered under the perfect pooling strategy. However, the intertemporal complexities make the results more difficult to establish. The time share in the first period could take on an intermediate value between the efficient shares for the two technology types; in other cases, the pooling time share might force overinvestment for all borrowers. The pooling interest rate always exceeds the complete information rate for type α firms. It could lie above or below the complete information rate for type β borrowers. It will be demonstrated that the lender's optimal pooling strategy always attracts too few type α firms into the market relative to the complete information contract;

the number of type β borrowers in the initial stage of lending could exceed or fall short of the complete information optimal level.

Once the outcome of the first-period projects are reported, the technological identity of some of the firms might be revealed unambiguously. Perfection binds the lender to offering these firms complete information contracts in the second period, qualified only by the number of first-period borrowers of the particular technology type.

Since it has been demonstrated that separating contracts always dominate pooling when only one loan is left to be negotiated, those firms that are not completely identified by their first-period projects are offered separating contracts in the last period. The lender has sufficient information to calculate the likelihood ratios for those project outcomes that could have been attained by firms with either type of technology. The information from the first period is incorporated by offering a particular firm a second-period loan contract that is dependent on the likelihood ratio, evaluated at the firm's first-period outcome.

The actual number of type α and type β firms that borrow under the terms of the contracts that result from a pooling strategy is critical in calculating the lender's total expected return. The determination of the number of initial borrowers is complicated by the fact that the first-period pooling contract parameters affect not only the expected profit in the first stage, but also the quantity constraints and likelihood functions used in evaluating the terms of the second-period contracts. In other words, the terms of the first-period contract might

be shifted from the optimal one-period pooling contract for two reasons. The lender might find it advantageous to relinquish some expected returns in the first period by lowering the interest rate in order to loosen the quantity constraints in the second period. In addition, the time share might be adjusted away from the allocation that maximizes first-period returns in order to improve the quality of information available for use in setting the second-period contracts.

To simplify the discussion, for the remaining portion of this subsection it will be assumed that the conditional distribution of the project return, $F_j(\theta_j)$, is uniform. Furthermore, it is assumed that the supports of θ_α and θ_β overlap over some range. Figure 11 illustrates such distributions. Then q_j is defined to be the probability of unambiguous identification for a firm of technology type j . These assumptions on the distribution functions restrict the number of different sets of separating contracts offered in the second period because the likelihood ratio used to determine the magnitude of the distortions in the contract parameters is held constant for all project outcomes in the region of overlap.

Firms with first-period outcomes in Region I are unambiguously revealed as type β firms; those in Region III are positively identified as type α firms. Recall that the probability of a positive outcome and the conditional distribution of θ_j , given a positive outcome, are independent by assumption. Hence, the probability of being perfectly identified for any given firm of technology j is given by

$$(9.12) \quad p_j(s_{p1})q_j, \quad j = \alpha, \beta.$$

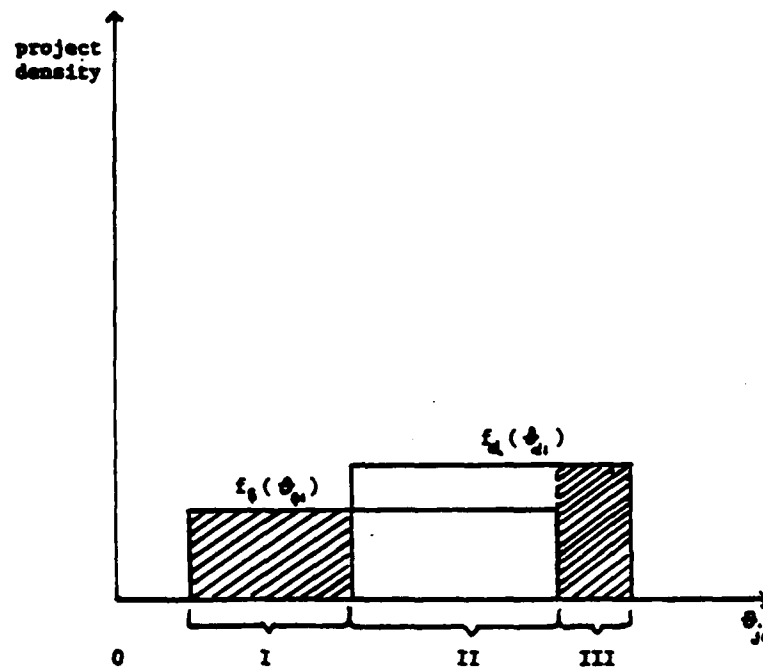


Figure 11: Project returns follow a uniform distribution.

Regions 0 and II are characterized by outcomes feasible for both technology types. The relevant likelihood ratio for Region 0 is the relative probability of total collapse for type β and type α firms under the terms of the first-period contract:

$$(9.13) \quad \frac{1 - p_{\beta}(\hat{s}_{p1})}{1 - p_{\alpha}(\hat{s}_{p1})}.$$

In Region II, the likelihood ratio used in determining the second-period contract terms is

$$9.14) \quad \frac{p_{\beta}(\hat{s}_{p1})(1 - q_{\beta})}{p_{\alpha}(\hat{s}_{p1})(1 - q_{\alpha})}.$$

The first step in the lender's selection of optimal contract parameters under a pooling strategy is the determination of the number of each type of borrower drawn into the market as a function of the contract terms. It will now be shown that the optimal contracts under the perfect pooling strategy can attract either too few firms of both technology types or too few type α entrepreneurs and too many type β firms. In no case will the lender's contract choices induce more than the complete information number of type α borrowers to enter the market.

The technological inefficiencies specified by the first-period contract could be great enough to reduce the number of firms of both the type α and the type β technologies below the complete information optimal quantities. However, a rational lender would never devise

contracts that attract more of both types of firms than the number of borrowers determined under the terms of the complete information contracts. The one-period analysis revealed that a pooling contract dictates inefficient production for both types of firms and charges an interest rate above the complete information type α interest rate. The terms of the first-period pooling contract in a two-period horizon might be adjusted away from these single-loan parameters in an effort to relax the second-period quantity restrictions facing the lender and move the parameters of subsequent contracts closer to the complete information optimum. However, because of the unavoidable trade-offs between technology types that must be confronted in any pooling strategy, it would never be profitable for the lender to introduce distortions of a magnitude sufficient to induce more than the desired number of both type α and type β borrowers. That is, if the first-stage parameters are set in such a way that too many entrepreneurs of one technology type choose to borrow, it must be true that too few of the other type secure loans.

The contracts offered under the pooling strategy could induce more than the complete information number of type β entrepreneurs to enter the market. For example, suppose that the first-period contract were at point A in Figure 12. Based on first-period considerations alone, too many type β borrowers and too few type α firms secure loans in the first period. In addition, the rewards required to elicit the cooperation of the type β borrowers in the second-period separating scheme can attract more than the desired number of these entrepreneurs even

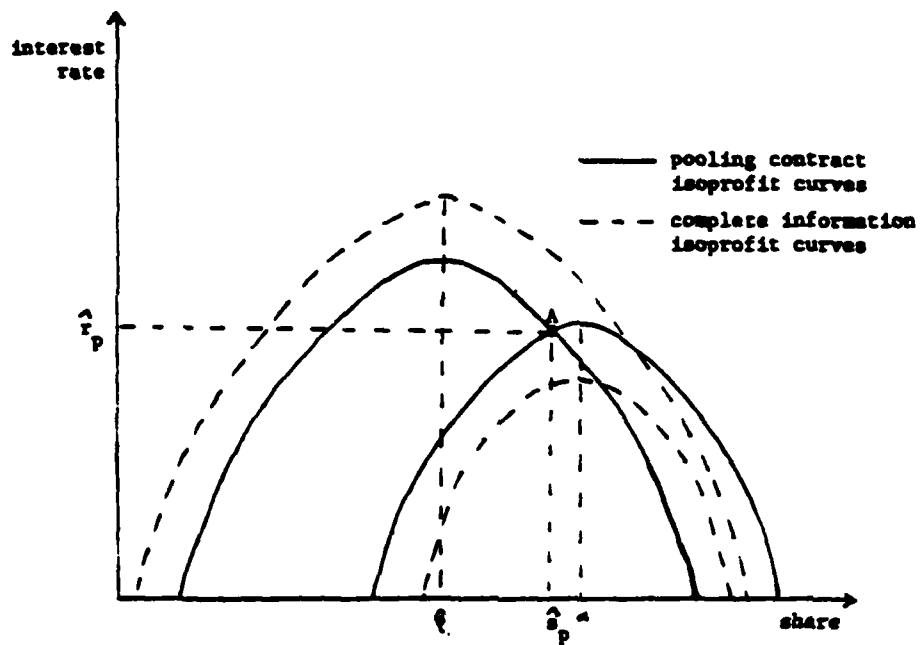


Figure 12: A pooling contract for the first round of a perfect strategy can induce too many type β firms to borrow.

when the first-period contract is less profitable than the complete information one.

In no case will a strategy that calls for pooling in the first period followed by separating contracts in the second attract too many type α firms and too few type β borrowers relative to the complete information optimum. Although this conclusion seems intuitively obvious, the supporting argument is complicated by the dependence of second-period contracts on first-period parameter choices. Any change in first-period terms that reduced the number of type α borrowers without decreasing the return to type β firms in the first period would necessarily involve a change in \hat{s}_{pl} . In turn, this adjustment leads to an alteration in the quality of information obtained from first-period project returns and available for constructing the second-period contracts. It must be established that the lender's efforts to reduce the number of type α borrowers to the desired level do not alter the information in such a way that even fewer type β borrowers choose to secure loans initially. In fact, it will be shown that any purported set of optimal contracts that attract too many type α but too few type β borrowers can be dominated by another set that decreases the number of type α firms to the complete information quantity in the initial period.

Suppose that the conjectured optimal pooling strategy were characterized by too many type α borrowers but too few type β firms in the market. The regularity conditions imposed on the isoprofit curves require that the pooling contract offered in the first period must lie

in the shaded region of Figure 13 at a point such as contract A. Perfection binds the lender to offer complete information contracts in the second period, subject only to potential quantity constraints, to those firms that are identified by their first-period project outcomes. Just as in the one-period analysis, the remaining separating contracts will be relatively less profitable for the type α firms and more profitable for the type β borrowers than the corresponding complete information contracts. Therefore, under the hypothesized optimal contracts, the marginal first-period type α borrower has a reservation wage given simply by the expected profit in the first period:

$$(9.15) \quad \gamma_{\alpha 1}^* = E\pi_{\alpha}(\hat{s}_{p1}, \hat{r}_{p1}) .$$

The type β borrower, on the other hand, can expect a higher profit on average in the second period than in the first because of the productive efficiency and low interest rate that characterize type β separating contracts. Therefore, the marginal borrower in this group has a reservation wage determined by:

$$(9.16) \quad \gamma_{\beta 1}^* = \frac{1}{2} \{ E\pi_{\beta}(\hat{s}_{p1}, \hat{r}_{p1}) + p_{\beta}(\hat{s}_{p1}) q_{\beta} E\pi_{\beta}(\bar{s}_{\beta 2}^I, \bar{r}_{\beta 2}^I) \\ + p_{\beta}(s_{p1})(1 - q_{\beta}) E\pi_{\beta}^{\text{II}}(\hat{s}_{\beta 2}, \hat{r}_{\beta 2}) + [1 - p_{\beta}(s_{p1})] E\pi_{\beta}^{\text{0}}(\hat{s}_{\beta 2}, \hat{r}_{\beta 2}) \} .$$

where the superscript denotes the region of the first-period outcome.^{21/} Maintaining the assumption that there are too many type α firms and too few type β entrepreneurs in the market, the isoprofit curves

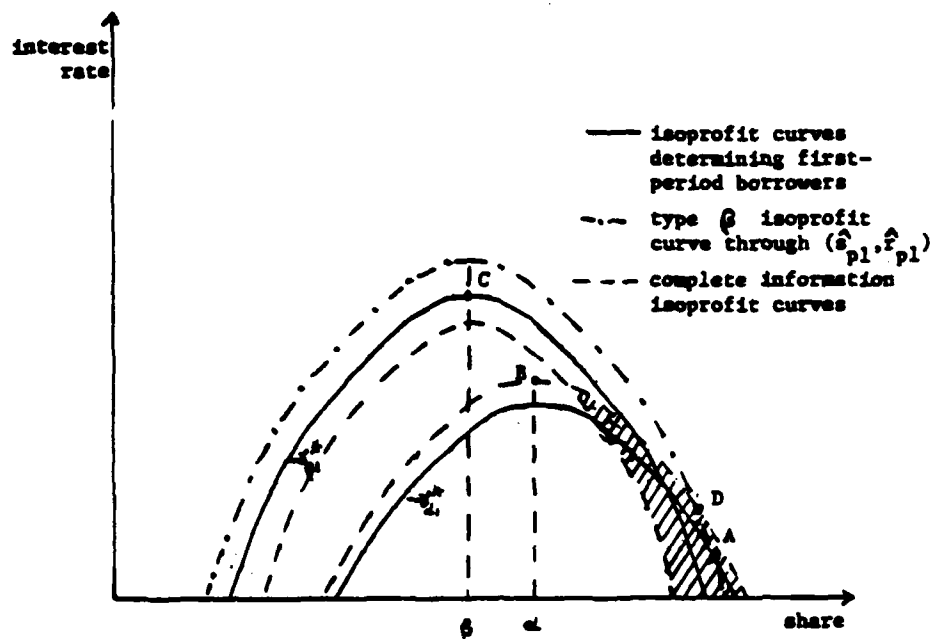


Figure 13: A pooling contract for the first round of a perfect strategy can never induce too many type α firms to borrow.

identifying the marginal borrower within each technological category are represented as $\gamma_{\alpha 1}^*$ and $\gamma_{\beta 1}^*$, respectively, in Figure 13.

In the second period, the complete information contract at point B will be offered to the $p_{\alpha}(\hat{s}_{pl})q_{\alpha}Q_{\alpha 1}$ identified type α firms. The remaining $[1 - p_{\alpha}(\hat{s}_{pl})q_{\alpha}]Q_{\alpha 1}$ type α borrowers are offered contracts to the northeast of point B, each of which is less profitable to both the lender and the entrepreneur than the contract at point B. Note that not all of these firms will choose to accept the second-period loans.

In order to meet the restrictions imposed by perfection, the type β contract offered to identified entrepreneurs in the second period must be set such that the marginal borrower in the first period is again indifferent between taking the complete information type β contract and exiting the market. The factor share is set at β and the interest rate is adjusted to the appropriate level. Point C represents this contract. All separating contracts extended to unidentified type β firms also permit productively efficient use of time; furthermore, they incorporate a reward in the form of a lower interest rate than that charged under contract C. Therefore, these contracts must be at least as profitable as the one at point C.

Given this hypothesized setting, the lender could increase its expected returns by moving the first-period contract from point A to point D in Figure 13. This pooling contract does not alter the profit to the type β borrowers in the initial period, but it decreases the number of type α borrowers to the complete information level. It will

be argued that such a move yields expected returns to the lender from each technology type that are at least as great as those anticipated under the conjectured optimal set of contracts.

Consider alterations in the lender's expected returns generated by moving the first-period pooling contract from point A to point D. Contract D is closer to the complete information type α contract, so the lender's return from firms of type α technology must increase in the first period as a result of this change. Similarly, the type β borrowers produce more efficiently at point D and pay a higher interest rate that leaves each entrepreneur's expected profit unchanged. Hence, the lender's return in the first period unambiguously improves by moving to a first-period contract that eliminates the excess type α borrowers but maintains the first-period profits of the type β firms.

No quantity restrictions have been introduced which bind the lender's choice of second-period type α contract parameters. Contract B is still offered to the identified type α entrepreneurs; the terms extended to the unidentified firms are altered only by the changes in the likelihood ratio induced by the change in \hat{s}_{p1} . However, since the first-period time allocation is more efficient under contract D than it was under contract A, more of the type α firms will be identified by their first-stage outcomes. Therefore, in the second period the lender must receive an expected return from these firms that is at least as large as the expected return that obtained under the original set of contracts.

The only contracts that could potentially decrease the lender's return under the proposed move from point A to point D are those offered to the type β firms in the second period. If the profit anticipated by these firms over the two-period horizon is diminished by the second-period contracts, even fewer of them will choose to enter the market initially. This, in turn, could decrease the lender's expected returns over the two-period horizon because of the stringent quantity restrictions in the second period. The alteration in \hat{s}_{p1} identifies more of the type β firms by the beginning of the second period. Therefore, fewer type β firms will be eligible for rewards under the second-period separating contracts. However, as long as the assumption of too few type β borrowers is maintained, the structure imposed by perfection ensures that the changes in second-period contracts induced by changes in \hat{s}_{p1} will not further diminish the number of type β entrepreneurs in the market. The contract offered to identified type β firms will specify efficient factor shares and an interest rate that just maintains the marginal borrower. Those firms that are not identified by the beginning of the second period earn an expected profit that is at least as high as the profit for the identified firms because of the reward necessary to accomplish separation. Therefore, it can be concluded that even though the likelihood ratios used in determining the parameters of the second-period separating contracts alter when the lender moves from contract A to contract D in the first period, there is no danger of causing even fewer type β firms to borrow in the initial stage.

The lender's first-period return clearly improves under the terms of contract D. Moreover, the changes in \hat{s}_{p1} do not diminish the average expected profit for type β firms; so the lender does not suffer a decrease in the total number of type β borrowers. Hence, one can conclude that no optimal pooling strategy will attract too many type α entrepreneurs and too few type β firms. Instead, any pooling scheme in the first period will attract either too few of both types of entrepreneurs or too few type α and too many type β borrowers relative to the complete information contracts.

Once the number of borrowers has been determined as a function of contract parameters, the lender's two-period problem is most easily approached by segmenting it according to time period and project outcome. At the beginning of the second stage, the likelihood ratios and the number of first-period borrowers have been determined. Treating these quantities as given, the lender chooses all second-period contract parameters optimally. Each set of contract terms applicable for a particular region of project outcomes can be determined independently of all other second-period contracts. The set of first-order conditions for these problems appears in the Appendix C.

Type α firms with first-period outcomes in Region III are offered complete information contracts. As discussed above, it is always true that too few type α firms actually borrow relative to the complete information optimum when the lender follows a pooling strategy. Therefore, none of the firms with outcomes in Region III will be eliminated by the second-period contract terms. As in the separating problem,

however, perfection requires the lender to devise the contracts so that the marginal firm is just indifferent between accepting the second loan and leaving the market. Thus, the lender's second-period problem for identified type α firms can be stated as:

$$\begin{aligned}
 (9.17) \quad E p^{III} = \max \{ p_{\alpha} (s_{\alpha 2}^{III}) [& \int_0^{1+r_{\alpha 2}^{III}} \theta_{\alpha 2} f_{\alpha}(\theta_{\alpha 2}) d\theta_{\alpha 2} \\
 & + \int_{1+r_{\alpha 2}^{III}}^{\infty} (1 + r_{\alpha 2}^{III}) f_{\alpha}(\theta_{\alpha 2}) d\theta_{\alpha 2} - (1 + i) \} Q_{\alpha 2}^{III} \\
 & + \mu_{\alpha 2}^{III} [L_{\alpha}^{III} G_{\alpha}(\gamma_{\alpha 2}^{III}) - Q_{\alpha 2}^{III}] + \tilde{\mu}_{\alpha 2}^{III} [L_{\alpha}^{III} Q_{\alpha 1} - Q_{\alpha 2}^{III}] \quad ,
 \end{aligned}$$

where L_{α}^{III} is the likelihood of realizing a first-period outcome in Region III. The first constraint on the number of borrowers in Region III is analogous to the quantity constraint in a single-loan problem. The lender must guarantee expected profits that are high enough to induce $Q_{\alpha 2}^{III}$ borrowers to remain in the market. The second quantity constraint reflects the fact that the number of loans extended in the second period is restricted by the number of borrowers in the first period.

The first-order conditions of the lender's expected return function lead to the anticipated conclusions. (See the Appendix C for a complete listing of these conditions.) The firms are allowed to produce efficiently. This is immediate from equation (C.2). However, when the lender is restricted by the initial number of type α borrowers, the

second-period interest rate is higher than the comparable rate in a one-loan complete information problem. Equation (C.3) supports this claim. Both quantity constraints are binding, so $\tilde{\mu}_{\alpha 2}^{III}$ is positive. Some simple substitution of first-order conditions reveals that the ratio $G_{\alpha}(\gamma_{\alpha 2}^{*III})/g_{\alpha}(\gamma_{\alpha 2}^{*III})$ is equated to only a fraction of the lender's marginal return from these firms. Hence, one can conclude that $\bar{r}_{\alpha 2}^{III}$ is higher than the optimal rate in a one-period complete information negotiation.

It was previously determined that, from the lender's viewpoint, the first-period pooling contract could attract too many type β borrowers when the pooling interest rate lies below \bar{r}_{β} . In that case, the second-period type β contract for firms with first-period outcomes in Region I are exactly the same as in the one-period complete information contract. The second quantity constraint for type β firms does not bind, so $\tilde{\mu}_{\beta 2}^I = 0$. As seen from equations (C.2) and (C.3), respectively, the time share is set at the efficient point and the interest rate equates the lender's marginal return with the ratio $G_{\beta}(\gamma_{\beta 2}^{*I})/g_{\beta}(\gamma_{\beta 2}^{*I})$.

The same conclusions do not hold when the pooling interest rate is sufficiently high. As long as \hat{r}_{p1} exceeds \bar{r}_{β} , the lender can conclusively determine that relatively too few type β entrepreneurs choose to borrow. The lender is constrained by the number of first-period type β borrowers even when \hat{r}_{p1} is set below \bar{r}_{β} if the optimal pooling contract requires gross productive inefficiencies for

the type β technology. In these cases, perfection forces a higher type β interest rate for firms that are unambiguously identified by their first-period project outcomes than the corresponding one-period complete information rate. Taking $\bar{\mu}_{\beta 2}^I$ as positive, this result follows through an argument analogous to the analysis of $\bar{r}_{\alpha 2}^{III}$ above.

The separating contracts devised for those firms with first-period outcomes in Regions 0 and II have all of the same qualitative properties as single-loan separating contracts. (The maximand and first-order conditions appear in Appendix C.) The type β firms produce efficiently (equation (C.13)) and pay a lower interest rate (equation (C.14)) than they would in a comparable complete information context, restricted to the same number of type β borrowers. The arguments used to establish these conclusions in Section 8 carry over without alteration. The magnitude of the distortions in the second period relative to the one-period separating contracts, however, depends on the likelihood ratios.

The optimal time allocation for type α firms under the terms of the second-period separating contracts is a monotonic function of the likelihood ratio of first-period outcomes, L_{β}^k/L_{α}^k . To see this, consider equation (C.8). The first-order conditions prove that $Q_{\alpha 2}^k$ is always a multiple of L_{α}^k ($k = 0, II$). Hence, that likelihood can be factored out of the denominator on the RHS of equation (C.8). With the exception of the likelihood ratio, all other terms in the equation are independent of \hat{s}_{p1} . Therefore, the RHS increases with the likelihood ratio. Since the LHS is increasing in $\hat{s}_{\alpha 2}^k$ for $\hat{s}_{\alpha 2}^k$ in the region above α , it follows

that $\hat{s}_{\alpha 2}^k$ increases monotonically with the likelihood ratio for $k = 0, II$.

The interest rate charged to the type α firms in the second period also increases with the likelihood ratio. Analysis of equation (C.9) proves this. Again, the likelihood $L_{\alpha 2}^k$ can be factored out of the LHS of equation (C.9) because $Q_{\alpha 2}^k$ is a multiple of L_{α}^k . Since L_{α}^k is always positive in the regions under consideration, one can divide by that term. Therefore, it is easy to see that the RHS decreases with the likelihood ratio L_{β}^k/L_{α}^k since all other terms in equation (C.9) are independent of \hat{s}_{p1} . The RHS is a decreasing function of the interest rate; thus, it follows that $\hat{r}_{\alpha 2}^k$ is a monotonically increasing function of the relevant likelihood ratio.

To adequately compensate the unidentified type β firms for their cooperation, the second-period type β interest rate decreases as the likelihood ratio decreases. When L_{β}^k is factored out of the LHS of equation (C.14), that condition is not directly dependent on \hat{s}_{p1} . However, the incentive constraint relevant for Region k requires

$$(9.18) \quad E\pi_{\beta}(\hat{s}_{\beta 2}^k, \hat{r}_{\beta 2}^k) \geq E\pi_{\beta}(\hat{s}_{\alpha 2}^k, \hat{r}_{\alpha 2}^k) .$$

It has just been determined that the type α contract becomes more profitable for the type α firms, and hence for the type β firms, in both dimensions as the likelihood ratio decreases. The type β separating contracts always specify efficient production. Therefore, in order to satisfy the incentive constraint, $\hat{r}_{\beta 2}^k$ must also decrease monotonically with the likelihood ratio.

If the expected number of firms with first-period outcomes in Region k is highly skewed toward type α firms, the type α time share is reduced toward α and the interest rate is lowered. The reward to the type β firms is correspondingly increased. This strategy allows the lender to reap higher total expected returns from the relatively large number of type α firms while facing a high payoff to a small expected number of type β firms. The distortions move in the opposite direction when type β firms are more likely to obtain an outcome in Region k . The lender reduces the reward to the large expected number of type β firms but suffers a substantial decrease in expected returns from the relatively few type α firms.

Note that the second-period expected profit for some unidentified type β firms might be higher than needed to maintain $Q_{\beta 2}^k$ firms in the market. Profit under the type β separating contract terms must be high enough to prevent these firms from taking the type α contract. In order to satisfy the incentive constraint, expected profit might be driven substantially higher than the amount required to meet the quantity constraint.

Having determined optimal second-period contract terms as a function of first-period parameters, the lender must select the first-period pooling contract terms. The optimal choice of these parameters involves some complicated decisions. Not only do these terms determine the expected return in the first period, but they affect returns in the second period in several ways. All firms must consider first-period expected profits when making their initial borrowing decisions.

Therefore, the number of firms in both the first and second periods are functions of the first-period contract parameters. In addition, the distribution of first-period project outcomes depends on the selection of \hat{s}_{p1} . Since all of the likelihood ratios are evaluated at the time allocation \hat{s}_{p1} , the magnitude of the distortions in the second-period separating contracts fundamentally depends on the choice of the first-period time share. When selecting the parameters of the first-period pooling contract, the lender must balance the value of expected returns in the first period with the benefit derived from better quality information available for use in constructing the second-period contracts.

It was previously determined that too few type α firms choose to borrow under the terms of any pooling strategy. Perfection requires that no second-period profit for type α firms exceed the amount required to maintain the marginal type α firm. In other words, first-period contract terms are all that are required to determine the number of expected type α entrepreneurs who actually borrow in the first period.

The incentive constraint required to separate unidentified firms in the second period might cause second-period expected profit for some type β firms to exceed expected profit under the terms of the first-period pooling contract. This could occur in spite of perfection. When this is the case, the expected number of initial type β borrowers is determined by the terms of both the first-period contract and the contract yielding the highest expected profit for type β firms in the second period. The likelihood ratios determine the relevant second-

period contract since the time share is always efficient and the interest rate adjusts with the likelihood ratio under the terms of the type β separating contract. As a matter of simplification, it will be assumed that the relevant second-period contract is the one for firms with project outcomes in Region II.

If no second-period type β contract leads to a higher expected profit than that of the first period, \hat{s}_{pl} and \hat{r}_{pl} are all that are needed to compute the expected number of type β borrowers in the first period. To cover all cases, the maximization will be done under two sets of constraints on the number of type β borrowers. Only one of these maximizations, however, is relevant at any given time.

First, suppose that no type β contract offered in the second period leads to higher expected profit than the first-period pooling contract for firms with type β technology. The lender's maximization problem can be stated as

$$(9.19) \quad E_p = \max_{j=\alpha, \beta} \left\{ p_j(s_{pl}) \left[\int_0^{1+r_{pl}} \theta_{j1} f_j(\theta_{j1}) d\theta_{j1} + \int_{1+r_{pl}}^{\infty} (1+r_{pl}) f_j(\theta_{j1}) d\theta_{j1} \right] - (1+i) Q_{j1} + \mu_{j1} [G_j(v_{j1}^*) - Q_{j1}] \right\} + \sum_{k=0}^{III} E_p^k,$$

where

$$(9.20) \quad v_{j1}^* = p_j(s_{pl}) \int_{1+r_{pl}}^{\infty} [\theta_{j1} - (1+r_{pl})] f_j(\theta_{j1}) d\theta_{j1}, \quad j = \alpha, \beta.$$

The first-order conditions pertaining to this function appear in Appendix C as equations (C.20) through (C.25).

As in the one-period pooling contract, few definitive conclusions can be reached about the pooling contract parameters used in the first period of a two-period negotiation. No restrictions are put on the bounds of the pooling factor share. However, the pooling interest rate will always exceed the complete information type α interest rate. It may, but need not, exceed the rate set for type β firms in a complete information context.

The pooling factor share plays a key role in determining the quality of information available to the lender in the second period. This relationship is captured in the A_j term in equation (C.20). Since A_j may be positive or negative, the information role of the pooling factor share could force overinvestment for all borrowers to increase or decrease relative to the pooling factor share in the one-period model.

The first-period pooling interest rate can be analyzed in the same manner as the one-period rate. Hence, it can be concluded that the pooling interest rate will always exceed \bar{r}_α , the complete information interest rate determined for type α firms. In some instances, even the complete information type β interest rate lies below the pooling rate.

Now suppose that the separating contract offered to type β firms who are unidentified in Region II in the second period promises higher expected profit for those firms than the first-period pooling contract.

This situation might arise when the expected number of type β firms in this region is so small relative to the number of type α firms that the lender would prefer to pay off the type β firms rather than severely distort the type α contract. Since each first-period type β borrower is equally likely to qualify for the highly profitable Region II contract in the second period, each type β entrepreneur's decision to enter the market is based on a weighted average of expected profits generated under both the pooling contract and the separating contract for Region II. The marginal type β borrower is the entrepreneur with reservation wage

$$(9.21) \quad \gamma_{\beta 1}^* = \frac{1}{1 + p_{\beta}(\hat{s}_{p1})(1 - q_{\beta})} \left\{ p_{\beta}(\hat{s}_{p1}) \int_{1+r_{p1}}^{\infty} [\theta_{\beta 1} - (1 + \hat{r}_{p1})] f_{\beta}(\theta_{\beta 1}) d\theta_{\beta 1} \right. \\ \left. + p_{\beta}(\hat{s}_{p1}) (1 - q_{\beta}) p_{\beta}(\hat{s}_{\beta 2}^{II}) \int_{1+r_{\beta 2}^{II}}^{\infty} [\theta_{\beta 2} - (1 + \hat{r}_{\beta 2}^{II})] f_{\beta}(\theta_{\beta 2}) d\theta_{\beta 2} \right\} .$$

When this quantity constraint is relevant, maximization of the lender's expected profit function leads to conditions (C.27) through (C.32) in the Appendix C.

Similar conclusions can be reached concerning the pooling interest rate under this scenario. However, now it is more likely that the pooling factor share will be used to discourage type β firms from entering the market in order to offset the attraction of the lucrative second-period contract in Region II. Relative to the previous case, \hat{s}_{p1} is now more likely to exceed α . If it is in the interval $[\beta, \alpha]$, it is pushed closer to the efficient allocation for the type α technology.

In summary, the main features of the lender's perfect pooling strategy in a two-loan problem can be described as follows:

The contracts offered when the lender chooses to pool all borrowers for the first stage of a two-loan horizon never induce more than the complete information number of type α entrepreneurs to secure loans. These contracts may, but need not always, attract more than the desired number of type β borrowers.

All firms that are identified by their first-period project returns are offered complete information contracts in the second period, modified only by the number of first-period borrowers of each type. Those firms that are not unambiguously categorized by the beginning of the second period are offered separating contracts. The distortions in the separating contract parameters are monotonic in the likelihood ratio for each project outcome. The more likely is a type j firm to realize a particular project outcome, the less severe the distortions will be for firms characterized by type j technology.

The pooling contract offered in the first period always requires an interest rate above the complete information type α interest rate to compensate the lender for the additional risk created by the inherent productive inefficiencies of the contract. No bounds are put on the possible values of the pooling time share. It could lie between the efficient shares for each technology type; alternatively, it could specify overinvestment in one activity for both of the technological groups.

The complexity of the relationship between the first- and second-period contract parameters precludes a simple characterization of the properties of the pooling interest rate and factor share. The lender could choose contract parameters that are far from the one-period optimal choice in order to improve its informational position in the second period. The trade-offs between the expected losses in one period and the expected gains in the other depend on the exogenous factors of the problem.

Results of a simulation study will be reported to demonstrate that, under some sets of exogenous parameters, a perfect strategy that entails an initial pooling contract can be the lender's optimal choice.

9.3 Pooling Contracts Are Not Dominated under Perfect Strategies

Even though a pooling strategy is never optimal for a single-loan transaction, a strategy that calls for pooling in the early periods of repeated transactions cannot be ruled out when the lender is restricted by perfection. This result is due to the fact that all of the type β firms in the market must be rewarded for revealing their identities in the initial stage of a separating strategy and forfeiting their informational advantage for all future transactions. Since the lender will gain some information simply by keeping track of the realized project outcomes, it could prove more profitable to wait until some information has been collected before attempting to separate the technological types completely. Some of the project returns are likely to unambiguously reveal firms with type β technology. These firms would no longer be eligible for separating payoffs. Moreover, the mix of technology types in the pool of unidentified firms could change in such a way that promotes a relatively low-cost separation procedure.

Figure 14 is suggestive of a case in which pooling at the outset of a two-period negotiation might not be dominated. Suppose that the lender attempts to separate technologies with the first contract offered. These contracts attract relatively too few type α entrepreneurs and too many type β borrowers. Perfection binds the lender to complete information contracts in the second period, adjusted

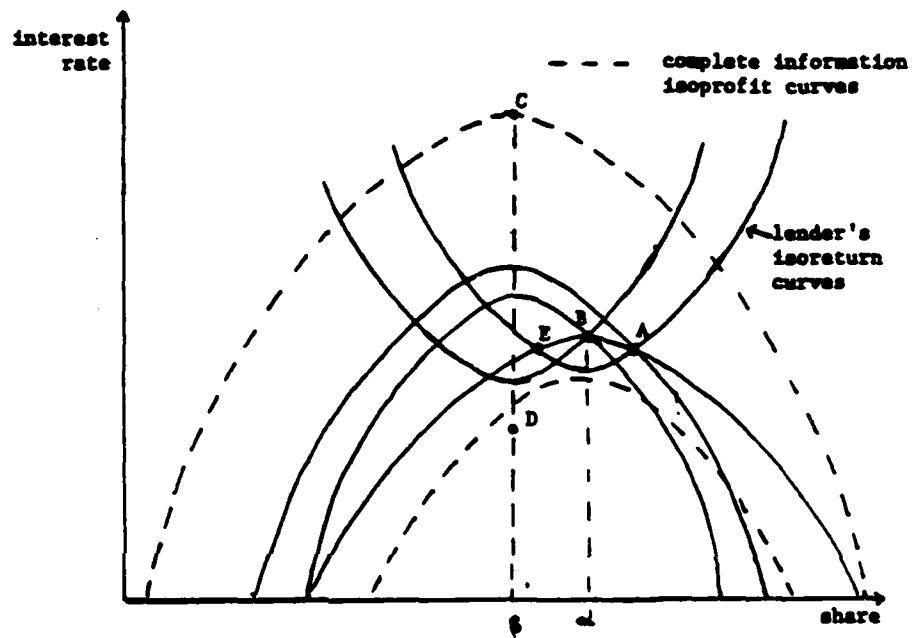


Figure 14: A pooling contract for the first round of a perfect strategy is not dominated by a pair of separating contracts.

for the number of first-period borrowers of each type. If point A represents the first-period type α contract, the second-period type α contract must be at point B. Point B specifies an efficient division of time and adjusts the interest rate upward so that the marginal type α borrower in the first period is again just indifferent between securing a second loan and leaving the market.

The second-period type β contract is at point C, which is identical to a one-period complete information contract. The time share is set efficiently, and no additional restrictions are put on the interest rate since more than the desired number of type β borrowers were attracted by the payoffs incorporated into the first-period contracts. In order to induce the cooperation of the type β firms in revealing their technological identities at the beginning of the initial period, the average of the expected profit under the two type β contracts must be at least as large as the average expected profit for firms with type β technology operating under the terms of the two type α contracts. In other words, the contract at point C must be averaged with a first-period contract with an interest rate sufficiently low so that the incentive constraint

$$(9.22) \quad E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) + E\pi_{\beta}(\bar{s}_{\beta 2}, \bar{r}_{\beta 2}) \geq E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + E\pi_{\beta}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2})$$

is satisfied. Point D illustrates such a contract.

Although the lender's expected return from the second-period type β contract is relatively high, the first-period contract necessary to induce separation is very costly to the lender. It might be better to

choose a pooling contract such as point E in the first period. By construction, this pooling contract leaves the lender's expected return from type α firms the same as under the first-period type α separating contract, but the expected return from the type β firms would be substantially increased. If a high proportion of firms are identified by the realizations of their first-period project returns, the loss from separating contracts in the second period might be relatively low. Hence, the pooling contract could prove to be superior to separating contracts in the initial period. This claim will be supported by numerical example.

The results of the numerical simulation are recorded in Tables 1 and 2. The example uses one of the simplest possible specifications of the model. Assume that, conditional on positive output, the random project outcome θ_α is uniformly distributed on the interval $[2,6]$; θ_β is uniformly distributed on $[0,5]$. This provides the motivation for wanting to identify the technology types. Even if the lender were assured of some positive outcome, on average the type β return would be lower. Hence, the lender would have to bear larger risk when dealing with these firms and would require compensation if it were possible to categorize borrowers a priori.

The functions determining the probability of positive output for each technology type are, respectively,

$$(9.23) \quad p_\alpha(s) = \frac{s^\alpha(1-s)^{1-\alpha} + \frac{1}{4}\alpha^\alpha(1-\alpha)^{1-\alpha}}{\frac{5}{4}\alpha^\alpha(1-\alpha)^{1-\alpha}}$$

and

$$(9.24) \quad p_{\beta}(s) = \frac{s^{\beta}(1-s)^{1-\beta} + \frac{1}{8}\beta^{\beta}(1-\beta)^{1-\beta}}{\frac{13}{8}\beta^{\beta}(1-\beta)^{1-\beta}} .$$

Each of these functions is constructed so that the probability of positive output is always greater than zero and attains its maximum when $s_j = j$. When each technology type is using the productively optimal time share, the probability of positive output is lower for type β firms than for type α firms, reflecting the more risky nature of the type β investment projects.

The measure of each type of entrepreneur who chooses to borrow is simply equal to the expected profit for that technology type under the terms of the contract set by the lender. As long as the expected profit is positive, some entrepreneurs will be willing to borrow funds.

Results are reported for two sets of exogenous parameters that differ only in the efficient time shares. In the first case, when α is set at .8 and β is set at .2, the lender's optimal strategy calls for separation from the outset. The great dispersion in the efficient time shares makes separation relatively inexpensive because a type β firm subjects itself to a very low probability of positive output under the type α contract. Hence, the required payoff in terms of a lower first-period type β interest rate is minimal.

In the second case, the lender's expected returns for the two-period horizon is maximized by offering a pooling contract in the first period, followed by separating contracts in the second period for those

Table 1: Numerical Example in which a Perfect
Separating Strategy is More
Profitable than Perfect Pooling

Parameters

$$\alpha = 0.8$$

$$\beta = 0.2$$

$$\bar{\theta}_{\alpha} = 6.0$$

$$\bar{\theta}_{\beta} = 5.0$$

$$\theta_{\alpha} = 2.0$$

$$\theta_{\beta} = 0.0$$

$$Q_{\alpha} = 1 \times \pi_{\alpha}$$

$$Q_{\beta} = 1 \times \pi_{\beta}$$

$$(1 + i) = 1.5$$

Complete Information Contract

Honest

$$\bar{s}_\alpha = 0.8$$

$$\bar{s}_\beta = 0.2$$

$$p_\alpha(\bar{s}_\alpha) = 0.9$$

$$p_\beta(\bar{s}_\beta) = 0.692$$

$$\bar{r}_\alpha = 2.945$$

$$\bar{r}_\beta = 3.709$$

$$Q_\alpha = 1.050$$

$$Q_\beta = 0.115$$

$$\pi_\alpha = 1.050$$

$$\pi_\beta = 0.115$$

$$\rho_\alpha = 1.050$$

$$\rho_\beta = 0.115$$

$$\rho_\alpha \times Q_\alpha = 1.103$$

$$\rho_\beta \times Q_\beta = 0.013$$

$$\sum \rho_j Q_j = 1.116$$

Complete Information Contract

All Take α Contract

$$\bar{s}_{\alpha} = 0.8$$

$$\bar{s}_{\beta} = 0.8$$

$$p_{\alpha}(\bar{s}_{\alpha}) = 0.9$$

$$p_{\beta}(\bar{s}_{\beta}) = 0.345$$

$$\bar{r}_{\alpha} = 2.945$$

$$\bar{r}_{\beta} = 2.945$$

$$Q_{\alpha} = 1.050$$

$$Q_{\beta} = 0.146$$

$$\pi_{\alpha} = 1.050$$

$$\pi_{\beta} = 0.146$$

$$\rho_{\alpha} = 1.050$$

$$\rho_{\beta} = -0.784$$

$$\rho_{\alpha} \times Q_{\alpha} = 1.103$$

$$\rho_{\beta} \times Q_{\beta} = -0.114$$

$$\sum \rho_j Q_j = 0.988$$

Perfect Separate — First Period

$$\hat{s}_{\alpha 1} = 0.802$$

$$\hat{s}_{\beta 1} = 0.2$$

$$p_{\alpha 1}(\hat{s}_{\alpha 1}) = 0.900$$

$$p_{\beta}(\hat{s}_{\beta 1}) = 0.692$$

$$\hat{r}_{\alpha 1} = 2.961$$

$$\hat{r}_{\beta 1} = 3.432$$

$$Q_{\alpha 1} = 1.039$$

$$Q_{\beta 1} = 0.170$$

$$\pi_{\alpha 1} = 1.039$$

$$\pi_{\beta 1} = 0.170$$

$$\rho_{\alpha 1} = 1.061$$

$$\rho_{\beta 1} = 0.060$$

$$\rho_{\alpha 1} \times Q_{\alpha 1} = 1.102$$

$$\rho_{\beta 1} \times Q_{\beta 1} = 0.010$$

$$\sum \rho_{j1} Q_{j1} = 1.113$$

Perfect Separate -- Second Period

$$\bar{s}_{\alpha 2} = 0.8$$

$$\bar{s}_{\beta 2} = 0.2$$

$$p_{\alpha}(\bar{s}_{\alpha 2}) = 0.9$$

$$p_{\beta}(\bar{s}_{\beta 2}) = 0.692$$

$$\bar{r}_{\alpha 2} = 2.962$$

$$\bar{r}_{\beta 2} = 3.709$$

$$Q_{\alpha 2} = 1.039$$

$$Q_{\beta 2} = 0.115$$

$$\pi_{\alpha 2} = 1.039$$

$$\pi_{\beta 2} = 0.115$$

$$\rho_{\alpha 2} = 1.061$$

$$\rho_{\beta 2} = 0.115$$

$$\rho_{\alpha 2} \times Q_{\alpha 2} = 1.102$$

$$\rho_{\beta 2} \times Q_{\beta 2} = 0.013$$

$$\sum \rho_{j2} Q_{j2} = 1.117$$

GRAND TOTAL FOR PERFECT SEPARATE = 2.228

Perfect Pool -- First Period

$$\hat{s}_{p1} = 0.801$$

$$\hat{s}_{p1} = 0.801$$

$$p_{\alpha}(\hat{s}_{p1}) = 0.900$$

$$p_{\beta}(\hat{s}_{p1}) = 0.344$$

$$\hat{r}_{p1} = 3.015$$

$$\hat{r}_{p1} = 3.015$$

$$Q_{\alpha 1} = 1.002$$

$$Q_{\beta 1} = 0.135$$

$$\pi_{\alpha 1} = 1.002$$

$$\pi_{\beta} = 0.135$$

$$\rho_{\alpha 1} = 1.098$$

$$\rho_{\beta 1} = -0.776$$

$$\rho_{\alpha 1} \times Q_{\alpha 1} = 1.100$$

$$\rho_{\beta 1} \times Q_{\beta 1} = -0.105$$

$$\sum \rho_{j1} Q_{j1} = 0.995$$

Perfect Pool -- Second Period
Identified in Region III and Region I

$$\bar{s}_{\alpha 2}^{-III} = 0.8$$

$$\bar{s}_{\beta 2}^{-I} = 0.2$$

$$p_{\alpha}(\bar{s}_{\alpha 2}^{-III}) = 0.9$$

$$p_{\beta}(\bar{s}_{\beta 2}^{-I}) = 0.692$$

$$\bar{r}_{\alpha 2}^{-III} = 3.01539$$

$$\bar{r}_{\beta 2}^{-I} = 3.709$$

$$Q_{\alpha 2}^{III} = 0.225$$

$$Q_{\beta 2}^I = 0.016$$

$$\pi_{\alpha 2}^{III} = 1.002$$

$$\pi_{\beta 2}^I = 0.115$$

$$\rho_{\alpha 2}^{III} = 1.098$$

$$\rho_{\beta 2}^I = 0.115$$

$$\rho_{\alpha 2}^{III} \times Q_{\alpha 2}^{III} = 0.247$$

$$\rho_{\beta 2}^I \times Q_{\beta 2}^I = 0.002$$

$$\rho_{\alpha 2}^{III} \times Q_{\alpha 2}^{III} + \rho_{\beta 2}^I \times Q_{\beta 2}^I = 0.249$$

Separate in Region II

$$\hat{s}_{\alpha 2}^{II} = 0.801$$

$$\hat{s}_{\beta 2}^{II} = 0.2$$

$$p_{\alpha}(\hat{s}_{\alpha 2}^{II}) = 0.899+$$

$$p_{\beta}(\hat{s}_{\beta 2}^{II}) = 0.692$$

$$\hat{r}_{\alpha 2}^{II} = 3.01541$$

$$\hat{r}_{\beta 2}^{II} = 3.601$$

$$q_{\alpha 2}^{II} = 0.676$$

$$q_{\beta 2}^{II} = 0.028$$

$$\pi_{\alpha 2}^{II} = 1.002$$

$$\pi_{\beta 2}^{II} = 0.135$$

$$\rho_{\alpha 2}^{II} = 1.098$$

$$\rho_{\beta 2}^{II} = 0.095$$

$$\rho_{\alpha 2}^{II} \times q_{\alpha 2}^{II} = 0.742$$

$$\rho_{\beta 2}^{II} \times q_{\beta 2}^{II} = 0.003$$

$$\sum \rho_{j2} q_{j2} = 0.745$$

Separate in Region 0

$$\hat{s}_{\alpha 2}^0 = 0.804$$

$$\hat{s}_{\beta 2}^0 = 0.2$$

$$p_{\alpha}(\hat{s}_{\alpha 2}^0) = 0.899$$

$$p_{\beta}(\hat{s}_{\beta 2}^0) = 0.692$$

$$\hat{r}_{\alpha 2}^0 = 3.015$$

$$\hat{r}_{\beta 2}^0 = 3.608$$

$$Q_{\alpha 2}^0 = 0.100$$

$$Q_{\beta 2}^0 = 0.088$$

$$\pi_{\alpha 2}^0 = 1.002$$

$$\pi_{\beta 2}^0 = 0.134$$

$$\rho_{\alpha 2}^0 = 1.098$$

$$\rho_{\beta 2}^0 = 0.097$$

$$\rho_{\alpha 2}^0 \times Q_{\alpha 2}^0 = 0.110$$

$$\rho_{\beta 2}^0 \times Q_{\beta 2}^0 = 0.009$$

$$\sum \rho_{j 2} Q_{j 2} = 0.119$$

GRAND TOTAL FOR PERFECT POOL = 2.108

Table 2: Numerical Example in which a Perfect
Pooling Strategy is More
Profitable than Perfect Separating

Parameters

$$\alpha = 0.5$$

$$\beta = 0.4$$

$$\theta_{\alpha} = 6.0$$

$$\theta_{\beta} = 5.0$$

$$\theta_{-\alpha} = 2.0$$

$$\theta_{-\beta} = 0.0$$

$$\theta_{\alpha} = \pi_{\alpha} \times 1$$

$$\theta_{\beta} = \pi_{\beta} \times 1$$

$$(1 + i) = 1.5$$

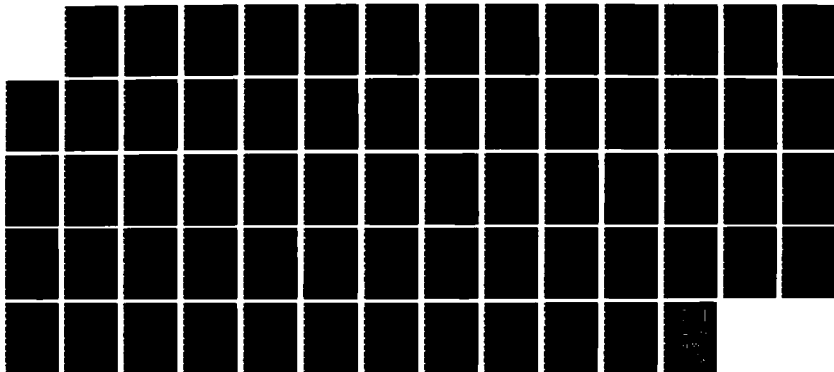
AD-A163 192

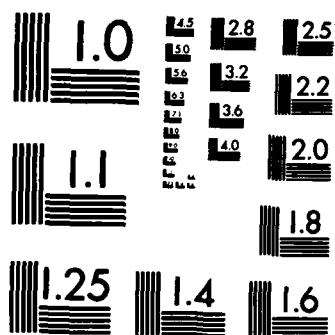
INTERACTIONS BETWEEN SIGNALING AND REPEATED PLAY WITH
BORROWER DEFAULT(U) STANFORD UNIV CA INST FOR
MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES C L SUCH
OCT 85 TR-480 N00014-79-C-0685 F/G 5/3

3/

UNCLASSIFIED

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Complete Information Contracts

Honest

$$\bar{s}_\alpha = 0.5$$

$$\bar{s}_\beta = 0.4$$

$$p_\alpha(\bar{s}_\alpha) = 0.9$$

$$p_\beta(\bar{s}_\beta) = 0.692$$

$$\bar{r}_\alpha = 2.945$$

$$\bar{r}_\beta = 3.709$$

$$Q_\alpha = 1.050$$

$$Q_\beta = 0.115$$

$$\pi_\alpha = 1.050$$

$$\pi_\beta = 0.115$$

$$\rho_\alpha = 1.050$$

$$\rho_\beta = 0.115$$

$$\rho_\alpha \times Q_\alpha = 1.103$$

$$\rho_\beta \times Q_\beta = 0.013$$

$$\sum p_j Q_j = 1.116$$

Complete Information Contract

All Take α Contract

$$\bar{s}_\alpha = 0.5$$

$$\bar{s}_\beta = 0.5$$

$$p_\alpha(\bar{s}_\alpha) = 0.9$$

$$p_\beta(\bar{s}_\beta) = 0.680$$

$$\bar{r}_\alpha = 2.945$$

$$\bar{r}_\beta = 2.945$$

$$Q_\alpha = 1.050$$

$$Q_\beta = 0.287$$

$$\pi_\alpha = 1.050$$

$$\pi_\beta = 0.287$$

$$\rho_\alpha = 1.050$$

$$\rho_\beta = -0.087$$

$$\rho_\alpha \times Q_\alpha = 1.103$$

$$\rho_\beta \times Q_\beta = -0.025$$

$$\sum \rho_j Q_j = 1.077$$

Perfect Separate -- First Period

$$\hat{s}_{\alpha 1} = 0.504$$

$$\hat{s}_{\beta 1} = 0.4$$

$$p_{\alpha}(\hat{s}_{\alpha 1}) = 0.899+$$

$$p_{\beta}(\hat{s}_{\beta 1}) = 0.692$$

$$\hat{r}_{\alpha 1} = 3.099$$

$$\hat{r}_{\beta 1} = 2.670$$

$$Q_{\alpha 1} = 0.947$$

$$Q_{\beta 1} = 0.376$$

$$\pi_{\alpha 1} = 0.947$$

$$\pi_{\beta 1} = 0.376$$

$$\rho_{\alpha 1} = 1.153$$

$$\rho_{\beta 1} = -0.145$$

$$\rho_{\alpha 1} \times Q_{\alpha 1} = 1.092$$

$$\rho_{\beta 1} \times Q_{\beta 1} = -0.055$$

$$\sum \rho_{j2} Q_{j2} = 1.037$$

Perfect Separate -- Second Period

$$\bar{s}_{\alpha 2} = 0.5$$

$$\bar{s}_{\beta 2} = 0.4$$

$$p_{\alpha}(\bar{s}_{\alpha 2}) = 0.9$$

$$p_{\beta}(\bar{s}_{\beta 2}) = 0.692$$

$$\bar{r}_{\alpha 2} = 3.099$$

$$\bar{r}_{\beta 2} = 3.709$$

$$Q_{\alpha 2} = 0.947$$

$$Q_{\beta 2} = 0.115$$

$$\pi_{\alpha 2} = 0.947$$

$$\pi_{\beta 2} = 0.115$$

$$\rho_{\alpha 2} = 1.153$$

$$\rho_{\beta 2} = 0.115$$

$$\rho_{\alpha 2} \times Q_{\alpha 2} = 1.092$$

$$\rho_{\beta 2} \times Q_{\beta 2} = 0.013$$

$$\sum \rho_{j2} Q_{j2} = 1.105$$

$$\text{GRAND TOTAL FOR PERFECT SEPARATE} = 2.142$$

Perfect Pool — First Period

$$\hat{s}_{p1} = 0.491$$

$$\hat{s}_{p1} = 0.491$$

$$p_{\alpha}(\hat{s}_{p1}) = 0.899$$

$$p_{\beta}(\hat{s}_{p1}) = 0.682$$

$$\hat{r}_{p1} = 3.018$$

$$\hat{r}_{p1} = 3.018$$

$$Q_{\alpha 1} = 1.000$$

$$Q_{\beta 1} = 0.268$$

$$\pi_{\alpha 1} = 1.000$$

$$\pi_{\beta 1} = 0.268$$

$$\rho_{\alpha 1} = 1.099$$

$$\rho_{\beta 1} = -0.063$$

$$\rho_{\alpha 1} \times Q_{\alpha 1} = 1.099$$

$$\rho_{\beta 1} \times Q_{\beta 1} = -0.017$$

$$\sum \rho_{j1} Q_{j1} = 1.083$$

Perfect Pool -- Second Period

Identified in Region III and Region I

$$\bar{s}_{\alpha 2}^{-III} = 0.5$$

$$\bar{s}_{\beta 2}^I = 0.4$$

$$p_{\alpha}(\bar{s}_{\alpha 2}^{-III}) = 0.9$$

$$p_{\beta}(\bar{s}_{\beta 2}^I) = 0.692$$

$$\bar{r}_{\alpha 2}^{-III} = 3.018$$

$$\bar{r}_{\beta 2}^I = 3.709$$

$$Q_{\alpha 2}^{III} = 0.225$$

$$Q_{\beta 2}^I = 0.031$$

$$\pi_{\alpha 2}^{III} = 1.000$$

$$\pi_{\beta 2}^I = 0.115$$

$$\rho_{\alpha 2}^{III} = 1.100$$

$$\rho_{\beta 2}^I = 0.115$$

$$\rho_{\alpha 2}^{III} \times Q_{\alpha 2}^{III} = 0.248$$

$$\rho_{\beta 2}^I \times Q_{\beta 2}^I = 0.004$$

$$\rho_{\alpha 2}^{III} \times Q_{\alpha 2}^{III} + \rho_{\beta 2}^I \times Q_{\beta 2}^I = 0.252$$

Separate in Region II

$$s_{\alpha 2}^{II} = 0.502$$

$$s_{\beta 2}^{II} = 0.400$$

$$p_{\alpha}(s_{\alpha 2}^{II}) = 0.899$$

$$p_{\beta}(s_{\alpha 2}^{II}) = 0.692$$

$$r_{\alpha 2}^{II} = 3.018$$

$$r_{\beta 2}^{II} = 3.037$$

$$q_{\alpha 2}^{II} = 0.674$$

$$q_{\beta 2}^{II} = 0.111$$

$$\pi_{\alpha 2}^{II} = 1.000$$

$$\pi_{\beta 2}^{II} = 0.267$$

$$\rho_{\alpha 2}^{II} = 1.100$$

$$\rho_{\beta 2}^{II} = - 0.036$$

$$\rho_{\alpha 2}^{II} \times q_{\alpha 2}^{II} = 0.741$$

$$\rho_{\beta 2}^{II} \times q_{\beta 2}^{II} = - 0.004$$

$$\sum \rho_{j 2}^{II} q_{j 2}^{II} = 0.737$$

Separate in Region 0

$$s_{\alpha 2}^0 = 0.505$$

$$s_{\beta 2}^0 = 0.4$$

$$p_{\alpha}(s_{\alpha 2}^0) = 0.899+$$

$$p_{\beta}(s_{\beta 2}^0) = 0.692$$

$$r_{\alpha 2}^0 = 3.173$$

$$r_{\beta 2}^0 = 3.191$$

$$q_{\alpha 2}^0 = 0.091$$

$$q_{\beta 2}^0 = 0.072$$

$$\pi_{\alpha 2}^0 = 0.899$$

$$\pi_{\beta 2}^0 = 0.226$$

$$\rho_{\alpha 2}^0 = 1.201$$

$$\rho_{\beta 2}^0 = 0.004$$

$$\rho_{\alpha 2}^0 \times q_{\alpha 2}^0 = 0.109$$

$$\rho_{\beta 2}^0 \times q_{\beta 2}^0 = 0.0003$$

$$\sum \rho_{j 2}^0 q_{j 2}^0 = 0.109$$

GRAND TOTAL FOR PERFECT POOL = 2.181

firms that remain unidentified by their first-period project outcomes. In this example, α is equal to .5 and β is set at .4; all other parameters remain unchanged. The difference in division of time is sufficiently small that relatively little productive efficiency is lost by a pooling contract. Moreover, the type α separating contract cannot discourage type β firms by gross overinvestment of time in one activity without also severely punishing the honest type α firms. Hence, a very low first-period interest rate for type β firms is required to accomplish separation from the beginning of the two-period span. This contract is particularly costly in terms of the large number of first-period type β borrowers that plan to exit the market rather than apply for a second loan. They are merely exploiting their informational superiority in order to take advantage of the low interest rate. In fact, separation is so costly that the lender would earn higher expected returns by simply offering all borrowers only the complete information contract intended for the type α technology in both periods without attempting to categorize the borrowers according to technological type.

The numerical example demonstrates that the results of a one-period transaction cannot be extrapolated to a multi-period setting. Sacrifices that would not be rational for a single-period transaction can become worthwhile for the lender if they permit higher returns in subsequent negotiations. Creditors frequently anticipate doing business with the same borrower over a long period of time. A significant proportion of the costs incurred with the initial loans stem from the lender's lack of familiarity with the borrower's methods of doing

business. Once this information has been effectively communicated, both parties can expect higher returns from future investment projects.

9.4 Imperfect Strategies in a Two-Period Model with Loan Size Fixed

If the lender's feasible strategies need not be perfect, the lender can make threats and promises in one stage of negotiations that bind its actions in another. The lender is no longer forced to maximize its remaining expected returns at each stage of contracting. The increased flexibility in the lender's strategy choices makes it easier to maintain standing relationships with the loan customers that do have positive value for the lender beyond the acquisition of improved information. The assumption that strategies need not be perfect is reasonable in contexts in which the lender's reputation for adhering to its bargains plays a role. For example, word-of-mouth communication among entrepreneurs is a vital conduit for ascertaining information about venture capitalists. In this manner, the venture capitalist's dealings with one firm can influence subsequent negotiations with other firms.

The possibility of employing imperfect strategies opens many options for the lender. These new options substantially alter the form of the equilibrium contracts. For example, the lender can now take advantage of the differences in reservation wages among borrowers of the same technological type and extract a higher interest rate from entrepreneurs with low opportunity costs. Since it is no longer constrained to maximize expected returns at each stage of negotiations, the lender can refuse to extend loans to some firms even after technological type

has been identified. This allows the lender to offer a low interest contract in the first period with the understanding that the firms that accept these contract terms will not be eligible for loans in the second period. Only entrepreneurs with high reservation wages would accept this deal; all other borrowers would agree to pay a higher interest rate in order to qualify for funds in later periods. In fact, the lender's optimal contract offerings will be characterized by price discrimination among type β borrowers but uniform treatment of all type α entrepreneurs.

Pooling contracts are dominated even at the outset of repeated transactions when imperfection is permitted. The separating contracts allow greater productive efficiency than a single pooling contract in the first period. In addition, rather than being bound to complete information contracts after identifying technological type in the first period, the lender can enhance the second-period type α contract. This proves to be a relatively inexpensive method of attracting more low-risk borrowers into the market.

9.5 Separating Contracts for the First Loan

The lender exercises a great deal of latitude in devising contract terms when it is uninhibited by the structure of perfection. Naturally, the wide variety of permissible contracts creates difficulties in specifying the lender's optimal strategy. For example, the lender might construct second-period contracts that are less than profit maximizing for that period alone, but which serve as an enticement to those firms that have a reasonable chance of qualifying for them. In this way the

lender might be able to mitigate the problem of too few type α borrowers under the terms of the first-period separating contract. This possibility could substantially increase the lender's expected return over the two periods, but it also complicates the determination of the initial number of borrowers.

Lifting the restriction of perfection does not affect some of the more general characteristics of the separating loan contracts. The first-period contracts still specify overinvestment and a higher interest rate than would obtain in a complete information context for type α firms. The firms with type β technology produce efficiently and are rewarded with a lower interest rate. The second-period type α contract yields expected profits for these firms that are at least as great as in the first period. However, a rational lender would never offer second-period contracts that raise the average expected profit over the two-loan horizon above the complete information level. Therefore, the lender knows conclusively that, relative to the complete information setting, too many type β firms and too few type α firms will choose to borrow in the first stage.

The lender still faces quantity constraints in the second period. However, it is no longer required to limit the expected profits of the firms so that the marginal borrower is no more than indifferent between securing the second loan and leaving the market. For example, it might prove worthwhile to make the type α contracts more profitable for those project outcomes that are unlikely to be attained by any type β firms. In so doing, the lender might be able to attract more of the

type α firms initially, thus relaxing the second-period quantity constraints, without having to pay a high reward to many type β firms.

A key distinction between this model and the variable-loan-size model of imperfect separating strategies is that the optimal second-period contracts for type β firms do depend on the first-period project outcomes. The variation in the reservation wage among borrowers is at the root of this difference. A type β entrepreneur with a high reservation wage might borrow the first time with the intention of applying for a second loan only if the firm's first-period project outcome qualifies it for a highly profitable second-period contract. The incentives of such a potential borrower have no effect on the less lucrative type β contracts offered in the second stage. The potential incentives for each entrepreneur need to be carefully analyzed in order to formulate the lender's maximand for an imperfect strategy.

Determining which of the possible incentive constraints are binding complicates the analysis of the lender's imperfect strategy in the most general case. Some of the flavor of the optimal contracts can be studied more easily by considering a very special case in which the first-period project outcome unambiguously identifies the technology type of each borrower. The incentives that motivate each entrepreneur depend on the individual's reservation wage, but the anticipated identification of technological type removes the uncertainty about the number of borrowers who participate in the second period.

For purposes of this portion of the analysis, the model will be slightly modified. Previously firms that experienced complete collapse

of their investment projects received a gross return of zero. Now it will be assumed that these firms realize a negligible positive return that just enables the lender to distinguish the technology type even when the investment project fails. This small return is denoted 2ϵ for the type α firms and ϵ for the type β borrowers.

When imperfection is permitted, the lender is able to take advantage of the known differences in reservation wages of the entrepreneurs. Separation on the opportunity costs of operating a small business must take the form of bribes to those entrepreneurs with high reservation wages. These firms are offered a lucrative contract for the first period with no possibility of obtaining a second loan. An entrepreneur who instead opts for a chance to borrow twice thereby identifies himself as having a relatively low reservation wage. The promise of a second loan allows the lender to charge these borrowers a higher interest rate without driving them out of the market or encouraging them to misrepresent their technological type. It will be shown that lender's optimal strategy calls for separating the type β firms on the basis of their individual reservation wages but treating all type α borrowers identically.

The general form of a separating scheme draws too few type α firms into the market relative to the complete information contracts. Since identification of individual opportunity costs requires forfeiting some business in the second period, the lender chooses not to attempt further segmentation of firms within the type α technological group (see Appendix E for a proof of this statement). Instead, the second-

period contract is used as a low-cost way of inducing more of these entrepreneurs to secure loans initially. The entry decision is based on the firm's expected profit averaged over the two periods. Therefore, the marginal type α borrower has a reservation wage defined by

$$(10.1) \quad \gamma_{\alpha 1}^* = \frac{1}{2} \sum_{t=1}^2 p_{\alpha}(s_{\alpha t}) \int_{1+r_{\alpha t}}^{\infty} [\theta_{\alpha t} - (1 + r_{\alpha t})] f_{\alpha}(\theta_{\alpha t}) d\theta_{\alpha t} .$$

Complete identification of technological type by the first-period project outcome obviates the need to deter type β firms from the second-period type α contract. Thus, the lender's least-cost method of increasing the number of type α borrowers calls for offering a more attractive contract in the second period.

The first-period type α contract requires overinvestment of time in one activity and a relatively high interest rate; the second-period contract is characterized by productive efficiency and a relatively low interest rate. When the lender is unimpaired by perfection, it can offer contract terms that leave all type α borrowers infra-marginal in the second period.

The payoffs necessary to induce separation of technological types attracts more than the desired number of type β entrepreneurs. The lender's total expenditure on pay-offs over the two-period horizon is minimized by offering type β firms a choice of contract policies (see Appendix C for a proof of this statement). The first option is a low-interest contract in the initial period with the stipulation that no subsequent loans will be extended. The marginal type β borrower that

selects this contract has a reservation wage just equal to the expected profit under the terms of this contract:

$$(10.2) \quad \gamma_{\beta H}^* = p_{\beta}(s_{\beta H}) \int_{1+r_{\beta H}}^{\infty} [\theta_{\beta H} - (1 + r_{\beta H})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} .$$

The second option is a pair of identical contracts with a higher interest rate but a guaranteed second-period loan. The entrepreneur who is just indifferent between the two type β contract offerings has a reservation wage defined by the equation

$$(10.3) \quad E\pi_{\beta}(\hat{s}_{\beta H}, \hat{r}_{\beta H}) + \gamma_{\beta L}^* = 2E\pi_{\beta}(\hat{s}_{\beta L}, \hat{r}_{\beta L}) .$$

The reward for revelation of technological type to those entrepreneurs who choose to borrow twice are distributed evenly over the two periods because of the convexity imposed on the profit functions.

The differences in reservation wages among type β entrepreneurs imposes different incentive constraints on the lender under the separating strategy. In order to prevent the type β entrepreneurs with high reservation wages from selecting a type α contract, the expected return under the first-period contract parameters must be at least as high as under the terms of the type α loan:

$$(10.4) \quad E\pi_{\beta}(\hat{s}_{\beta H}, \hat{r}_{\beta H}) \geq E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) .$$

Once this condition is met, however, no further constraints need be imposed on the contract offerings in order to achieve separation of technological types. The type β firms with low opportunity costs prefer the two-contract option to the one-period type β contract.

Knowing that their first-period project outcome will prevent them from securing a second-period type α contract, satisfaction of inequality (10.4) assures honest disclosure on the part of all type β borrowers.

Having determined the binding incentive constraint on the policy offerings in the special case in which technologies are identified by the end of the first period, the lender's maximand for a separating strategy can be stated as

$$\begin{aligned}
 (10.5) \quad E_p = & \sum_{t=1}^2 \{ p_{\alpha}(s_{\alpha t}) \left[\int_0^{1+r_{\alpha t}} \theta_{\alpha t} f_{\alpha}(\theta_{\alpha t}) d\theta_{\alpha t} + \int_{1+r_{\alpha t}}^{\infty} (1+r_{\alpha t}) f_{\alpha}(\theta_{\alpha t}) d\theta_{\alpha t} \right] \right. \\
 & - (1+i) + [1 - p_{\alpha}(s_{\alpha t})] 2\epsilon \} Q_{\alpha t} + \mu_{\alpha t} [G_{\alpha}(\gamma_{\alpha t}^*) - Q_{\alpha t}] \\
 & + \tilde{\mu}_{\alpha 2} [Q_{\alpha 1} - Q_{\alpha 2}] \\
 & + 2 \{ p_{\beta}(s_{\beta L}) \left[\int_0^{1+r_{\beta L}} \theta_{\beta L} f_{\beta}(\theta_{\beta L}) d\theta_{\beta L} + \int_{1+r_{\beta L}}^{\infty} (1+r_{\beta L}) f_{\beta}(\theta_{\beta L}) d\theta_{\beta L} \right] \right. \\
 & - (1+i) + [1 - p_{\beta}(s_{\beta L})] \epsilon \} Q_{\beta L} + \mu_{\beta L} [G_{\beta}(\gamma_{\beta}^*) - Q_{\beta L}] \\
 & + \{ p_{\beta}(s_{\beta H}) \left[\int_0^{1+r_{\beta H}} \theta_{\beta H} f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} + \int_{1+r_{\beta H}}^{\infty} (1+r_{\beta H}) f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} \right] \right. \\
 & - (1+i) + [1 - p_{\beta}(s_{\beta H})] \epsilon \} Q_{\beta H} + \mu_{\beta H} [G_{\beta}(\gamma_{\beta H}^*) - G_{\beta}(\gamma_{\beta L}^*) - Q_{\beta H}] \\
 & + \lambda \{ p_{\beta}(s_{\beta H}) \int_{1+r_{\beta H}}^{\infty} [\theta_{\beta H} - (1+r_{\beta H})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} \\
 & - p_{\beta}(s_{\alpha 1}) \int_{1+r_{\alpha 1}}^{\infty} [\theta_{\beta H} - (1+r_{\alpha 1})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} \} .
 \end{aligned}$$

where each of the cut-off reservation wages is defined above. The first-order conditions for this function are set out in Appendix D.

The time allocation is set above the productively efficient point in the first period for type α firms. This can be seen from equation (D.2) using arguments analogous to those in the single-loan separating model. In the second period, however, no distortion of the factor share is necessary. This follows directly from equation (D.6). All firms have been unambiguously identified by their project outcomes before the second-period contracts are offered; consequently, there is no need to deter dishonest type β borrowers with productive inefficiency under the terms of the second-period type α contract.

Just as in the one-loan problem, the type α interest rate is higher in the first period than the complete information interest rate. This claim is supported by equation (D.3). Again, the proofs from the single-loan model apply without alteration. Imperfection makes the determination of the second-period interest rate more difficult. Condition (D.7) can be reduced to

$$(10.6) \quad Q_{\alpha} \leq \frac{1}{2} \mu_{\alpha 1} g_{\alpha}(\gamma_{\alpha 1}^*) .$$

Equation (D.3) reveals that this condition holds with strict inequality. The optimal contracts for the two-period horizon are suboptimal when viewed individually. Once the second period is reached, the lender could increase its remaining expected return by raising the interest rate charged to type α borrowers. However, fewer type α entrepreneurs would apply for loans initially if they expected the lender to

renege on its commitments. The ability to make credible promises of low interest rates in the second period enables the lender to attract a larger number of type α borrowers and thereby gain a higher return over the two-period horizon.

All separating contracts offered to type β firms permit an efficient use of time. Equations (D.11) and (D.15) verify this. No deterrence needs to be incorporated into these contracts, so productive efficiency is optimal for both the lender and the borrower.

Both type β contracts offered in the first period charge an interest rate lower than the complete information rate. The argument that establishes this conclusion for the contract selected by entrepreneurs with high reservation wages is analogous to the one-period model. Substitution of the quantity constraints into equation (D.16) yields

$$(10.7) \quad G_{\beta}(\gamma_{\beta H}^*) - E p_{\beta}(\hat{s}_{\beta H}, \hat{r}_{\beta H}) g_{\beta}(\gamma_{\beta H}^*) = \lambda .$$

If the lender could costlessly identify firm technology, the interest rate would adjust until the LHS of equation (10.7) was set to zero. The Lagrange multiplier for the incentive constraint is positive, and the LHS is decreasing in the interest rate. Therefore, it can be concluded that $\hat{r}_{\beta H} > \bar{r}_{\beta}$.

By construction, the type β contract that permits borrowing in both periods yields a higher expected return to the entrepreneur in the first period than the other type β alternative. Both contracts are characterized by productive efficiency, so it follows that $\hat{r}_{\beta L} > \hat{r}_{\beta H}$. Nevertheless, the interest rate for the two-period contract is still

below the complete information type β interest rate. After substituting quantity constraints into the first-order condition with respect to $\hat{r}_{\beta 1}$, equation (D.12) can be rewritten as

$$(10.8) \quad G_{\beta}(\gamma_{\beta L}^*) = [2E\rho_{\beta}(\hat{s}_{\beta L}, \hat{r}_{\beta L}) - E\rho_{\beta}(\hat{s}_{\beta H}, \hat{r}_{\beta H})]g_{\beta}(\gamma_{\beta L}^*) .$$

Because $\hat{s}_{\beta L} = \hat{s}_{\beta H} = \beta$ and $\hat{r}_{\beta L} > \hat{r}_{\beta H}$, it follows that

$$(10.9) \quad G_{\beta}(\gamma_{\beta L}^*) > E\rho_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1})g_{\beta}(\gamma_{\beta 1}^*) .$$

These two expressions would be equated in a complete information context. Hence, it can be concluded that even the less profitable of the two type β contracts charges an interest rate below that which would obtain if the lender knew each firm's technology type from the outset. The type β entrepreneurs with lower opportunity costs still inflict an externality in the credit market. Therefore, they must be rewarded with a lower interest rate in order to induce their cooperation with the separation scheme.

The ability to enforce threats and promises allows the lender to discriminate on the basis of intrinsic personal characteristics among entrepreneurs within a technological group. This flexibility reduces the total amount of payments to type β firms required to accomplish separation of technological categories. The lowest interest rate available to type β firms in the first period determines the highest opportunity cost of doing business. However, the threat of refusing funds for a second period limits the number of entrepreneurs who apply for this lucrative contract. Instead, those type β entrepreneurs with

relatively low opportunity costs are willing to pay a higher interest rate in return for the assurance of qualifying for a second loan.

Although the lender does not find it profitable to try to discriminate among the type α entrepreneurs, imperfection still improves the lender's return from loans extended to this group. Too few of these entrepreneurs choose to borrow under the terms of a separating scheme. The lender can relax this quantity constraint without increasing the payments to type β firms by promising a type α contract with a low interest rate in the second period. This contract does not maximize the lender's return in the second period alone, but it does increase the average expected return over the two periods.

Imperfection significantly transforms the form of the optimal contracts when the lender chooses to separate technological types at the outset of repeated transactions. The properties of these contracts can be summarized as follows:

A lender empowered with imperfect strategies can discriminate among borrowers of the same technological type because of the individual differences in the opportunity costs of doing business. Discrimination must take the form of a low interest rate contract in the first period but refusal to do business in the second period. Since all separating schemes attract too few low-risk type α firms and too many type β borrowers, the lender chooses to treat all type α firms alike over the two-period horizon but offers two different sets of contracts to type β borrowers. Those type β firms with low reservation wages are willing to pay a higher interest rate in the first period in order to qualify for another loan in the second period. The discrimination among the high-risk type β borrowers minimizes the total expenditure on rewards necessary to accomplish separation of technological categories.

In the first period, the type α contract requires overinvestment in the signal and too high an interest rate relative to the complete information rate. In the second period, however, no adverse incentive effects need be considered. Hence, the lender can offer a lucrative contract to help increase the number of type α borrowers in the initial stage of contracting. All contracts for type β firms are characterized by an efficient time share. Those entrepreneurs with high reservation wages apply for a low interest loan in the first-period but leave the market in the second period. Those type β entrepreneurs with lower reservation wages agree to a higher interest rate payment in return for the guarantee of funds in the second period. The rewards for cooperation paid to these firms are distributed evenly over the two loan contracts.

The contracts devised under imperfect separating strategies have been analyzed only for the special case in which the first-period project outcome unambiguously identifies each firm's technological type. In a more realistic context in which uncertainty about technology was not completely resolved after the first round of negotiations, the optimal contracts would retain much of the flavor of those analyzed above. The lender would still choose to employ price discrimination among type β borrowers to minimize the expenditure on rewards. The type α contracts would still require productive distortions in the first period and allow for more lucrative returns to identified firms in the second stage. However, determination of the binding incentive constraints in this context proves to be an extremely complex problem. Second-period contract terms for unidentified firms are dependent on the likelihood ratios evaluated at the level of the first-period project outcome. Hence, each type β entrepreneur's cooperation with the separation scheme depends on both the probability of qualifying for the more profitable second-period contracts and on the individual

reservation wage. Several different incentive constraints are imposed on the lender due to the differences among borrowers within the type β technological category.

9.5 Pooling Contracts for the First Loan

Pooling contracts can be ruled out even when multiple transactions are anticipated, provided that the lender is allowed to follow imperfect strategies. To understand this conclusion, suppose that the lender chooses to pool in the first period. When pooling contracts are offered, the lender's strategy entails no commitments that bind the terms of the second loans. Hence, optimal separating contracts will be offered in the second stage. The lender could accomplish exactly the same results in the second period but improve its expected return from the loans offered in the first period. Just as in the one-loan problem, the pooling contract involves inherent productive inefficiencies for both technology types. These inefficiencies could be reduced. Some pair of separating contracts could be constructed that provides the same expected profits for each type of firm but improves the expected returns for the lender. Since all forms of commitments are credible when imperfection is permitted, the lender's second-period decisions need not be bound by any of the information gained from the first period. Therefore, the lender strictly improves its total expected returns over the two-loan horizon by separating firms at both stages.

11. Conclusion

The contracts analyzed in the preceding sections are those that obtain when a lender with market power conducts business with start-up firms over a two-period horizon. Because a second round of negotiations is anticipated, the first-period decisions are complicated by intertemporal trade-offs, contract terms that are less than profit maximizing for the first period alone might be offered initially because the information gained from the project returns generated under these terms allows the lender to more than recoup the difference in returns in the second period. The number of repetitions is finite, however; so the lender finds it useful to employ a screen to help distinguish risk classes of borrowers and offer contracts tailored more nearly to each firm's technology. The effect of repetition on the use of the signal has been the focus of interest throughout the study.

Many of the features of the equilibrium contracts and the implicit assumptions used in the analysis are observed in the negotiations between a venture capitalist and its clients. Because of the high degree of risk inherent in venture capital financing, the creditor takes an active part in supervising the business plan of the new firm. Although the venture capitalist's experience of financing other start-up firms in the past enables it to judge a new firm's creditworthiness against an established standard, the entrepreneur may have knowledge that is peculiar to its firm which would warrant deviations in the business plan from the industry norm. These asymmetries in information, and consequent conflicts of interest, between the two contracting

parties can help to explain characteristics of venture capital financing which violate the standard competitive model.

The lender in the foregoing models exercises monopoly power over the availability of funds to the start-up firms. Although this assumption oversimplifies the credit transactions of start-up firms, the venture capitalist does wield a considerable degree of market power. Typically, venture capitalists specialize their expertise by field of technological innovation, stage of development of the firm, and geographic location. Hence, the numbers of potential financiers available to a particular entrepreneur is limited. In the initial stages of their relationship, the venture capitalist devotes considerable time and resources to the assessment of the business practices and projected profitability of its client. Since this evaluation process is costly, the information is not readily transferred to a second venture capitalist. As a result, the entrepreneur is, in a very real sense, bound to the financier with whom he originally transacted business.

One of the biggest complaints voiced by venture capitalists is that entrepreneurs in high technology industries are primarily interested in the development of the engineering aspects of their product line and pay too little attention to marketing and growth potential. To capture this notion, the lender uses the allocation of resources between engineering and other business activities proposed in the business plan as a proxy for the true risk associated with the financing of the start-up firm. The signal works because the degree of risk inherent in each investment project is inversely correlated with the proportion of

resources devoted to strictly engineering tasks. Each entrepreneur is assumed to know the most efficient allocation for his particular technology, but he may be willing to distort the use of resources in order to convince the venture capitalist that he operates a creditworthy business.

The lender in both models finds it profitable to offer loan contracts that separate the risk classes when only one transaction is anticipated. The high-risk borrowers inflict a negative externality on the low-risk ones in the market. This externality manifests itself in the form of less than maximal productive efficiency and a higher interest rate for all low-risk entrepreneurs. In addition, the high-risk firms' cooperation in identifying their true technology is encouraged by contract parameters that specify productive efficiency and a lower interest rate than would obtain if the lender could costlessly classify all borrowers before granting loan terms.

The pooling contract, which forces all borrowers to use the same resource allocation and pay the same interest rate, involves productive inefficiencies for all borrowers. For this reason, the pooling contract is dominated by a pair of separating contracts when only one round of negotiations is possible. However, if the lender is restricted to only credible threats and promises, pooling contracts may be more profitable than separating ones in the early stages of multiple transactions. Once the lender has classified firms by their technological types, the only credible contract offerings are complete information ones. Realizing that identification means forfeiting all future low-risk contracts, the

high-risk firms require a reward commensurate with their expected foregone earnings in return for identifying their technological category before the last round of financing. Moreover, the lender learns something about its borrowers simply by observing their project returns at each stage. The lender incorporates this information by making subsequent contract terms depend on the likelihood ratio for each project outcome in the previous stages. The high cost of separating firms initially and the ability to modify contract terms in later stages by the information learned from observing realized outcomes make a pooling contract a viable alternative to separating ones at the outset of negotiations restricted by perfection.

The phenomenon of overinvestment in one input is not unique to the separating contracts derived in the models. The separating contract offered to the low-risk firms always prescribes overinvestment in order to convince the lender of the borrowers' true technology type. However, in some circumstances the pooling contract can require overinvestment for all borrowers. When the pooling contract is optimal but one class of firms will generate large expected losses for the lender under the contract terms, the lender specifies a severe misallocation of resources for these borrowers in order to minimize the size of the loan demand from them. Although the other class of borrowers also suffers inefficiencies in production, the distortions are of a much smaller magnitude for their technology type.

The degree of control permitted the lender is instrumental in determining the form of the equilibrium contracts. When perfection is

required, the lender might find it useful to extract information about each borrower's technological identity; but there is no way to distinguish borrowers within a single category. The only way that the lender could further segment the market is to offer a highly profitable contract in the initial stage of financing but refuse to do business in the second round. However, once the firm has revealed its technology, the lender's threat to withhold funds is no longer credible.

In contrast, when the more powerful imperfect strategies are allowed, the lender can make use of the differences among the high-risk borrowers to reduce the amount of the reward needed to separate the technological groups. By inducing the high-risk borrowers with high reservation wages to leave the market after the initial round of financing, the payments made to the remaining high-risk firms are decreased. Although the lender could potentially elicit more information about the borrowers within the low-risk technological category, the costs outweigh the benefits of extraction.

The value of long-standing relationships between a creditor and its borrowers is apparent from observation of the development of credit institutions. The externality inflicted on the low-risk firms is mitigated to some extent simply by keeping detailed credit histories of all borrowers. However, the lender's reputation also plays an important role in promoting market efficiency. When the lender is willing to enforce a threat even though it leads to a short-run loss, an example is set for all future borrowers. Consequently, in future periods the

externality imposed on the low-risk firms is attenuated by the influence of the lender's threats on the behavior of the high-risk firms.

Any attempt to assess social welfare in venture capital markets must weigh the influence of a strong reputation in reducing the problems created by incomplete information against the losses generated by significant market power. Information pertinent to acquiring financing is commonly transmitted from one cohort of start-up firms to the next. Hence, the venture capitalist's reputation for fair dealing and willingness to discipline self-serving entrepreneurs is important in molding the framework of the relationship between the two parties, and, ultimately, the institutions of the market. The ability to exercise this power obviously benefits the venture capitalist. It also helps to minimize the cost of identifying the low-risk firms, thereby permitting an allocation of funds more similar to the complete information allocation than would be possible without the venture capitalist's ability to exercise strong market power. Consequently, the social inefficiencies associated with market power in certainty models might be mitigated to some extent when information is incomplete.

Appendices

Introduction to Appendices A and B

Throughout Appendices A and B, the following notation is used:

$$A_j = \gamma_j^2 [E(\theta_j | \theta_j > \theta_j^*)]^2, \quad j = \alpha, \beta$$

$$R_j = \frac{F_j(\theta_j^*)E(\theta_j | \theta_j < \theta_j^*) + \frac{1}{2} [1 - F_j(\theta_j^*)]E(\theta_j | \theta_j > \theta_j^*)}{E(\theta_j | \theta_j > \theta_j^*)}, \quad j = \alpha, \beta$$

$$C(r_{..}) = \frac{1 + i}{1 + r_{..}}$$

$$E\rho_j(s_{..}, r_{..}) = \frac{A_j s_{..}^{2j} (1 - s_{..})^{1-2j}}{2(1 + \hat{r}_{..})} [R_j - \frac{1}{2}C(r_{..})]$$

$$E\pi_j(s_{..}, r_{..}) = \frac{A_j s_{..}^{2j} (1 - s_{..})^{1-2j}}{4(1 + r_{..})} [1 - F_j(\theta_j^*)]$$

$$U_j(\omega) = g_j(\omega) \omega E\rho_j(s_{.2}(\omega), r_{.2}(\omega))$$

$$V_j(\omega) = g_j(\omega) \omega E\pi_j(s_{.2}(\omega), r_{.2}(\omega))$$

Appendix A

Perfect Strategy--Pool for the First Loan

Maximand

$$\begin{aligned}
 (A.1) \quad E_p = \max_{j=\alpha, \beta} \quad & \sum_j N_j [E\rho_j(s_{p1}, r_{p1}) + p_j E\rho_j(\bar{s}_{j2}, \bar{r}_{j2})] \\
 & + \int_Q \delta[g_\alpha(\omega), g_\beta(\omega)] \left\{ \sum_{j=\alpha, \beta} g_j(\omega) N_j E\rho_j(s_{p2}(\omega), r_{p2}(\omega)) \right\} d\omega \\
 & + \int_Q \{1 - \delta[g_\alpha(\omega), g_\beta(\omega)]\} \left\{ \sum_{j=\alpha, \beta} g_j(\omega) N_j E\rho_j(s_{j2}(\omega), r_{j2}(\omega)) \right. \\
 & \left. + \lambda(\omega) g_\beta N_\beta [E\pi_\beta(s_{\beta2}(\omega), r_{\beta2}(\omega)) - E\pi_\beta(s_{\alpha2}(\omega), r_{\alpha2}(\omega))] \right\} d\omega
 \end{aligned}$$

First Order Conditions

$$\begin{aligned}
 (A.2) \quad \frac{\partial E\rho}{\partial r_{p1}} = 0 &\Rightarrow C(\hat{r}_{p1}) \sum_{j=\alpha, \beta} N_j A_j \hat{s}_{p1}^{2j} (1 - \hat{s}_{p1})^{1-2j} \\
 &= \sum_{j=\alpha, \beta} N_j \{A_j R_j \hat{s}_{p1}^{2j} (1 - \hat{s}_{p1})^{1-2j} \\
 &\quad - (1 + \hat{r}_{p1}) \int_{\Omega} \frac{\partial g_j(\omega)}{\partial r_{p1}} E\rho_j(\hat{s}_{j2}(\omega), \hat{r}_{j2}(\omega)) d\omega \\
 &\quad + [U_j(\bar{\omega}) - U_j(\underline{\omega})]\}
 \end{aligned}$$

$$\begin{aligned}
 (A.3) \quad \frac{\partial E\rho}{\partial s_{p1}} = 0 &\Rightarrow \sum_{j=\alpha, \beta} N_j \{E\rho_j(\hat{s}_{p1}, \hat{r}_{p1})(2j - \hat{s}_{p1}) \\
 &\quad + \hat{s}_{p1}(1 - \hat{s}_{p1}) \frac{\partial p_j}{\partial s_{p1}} E\rho_j(\bar{s}_{j2}, \bar{r}_{j2}) \\
 &\quad + \int_{\Omega} \frac{\partial g_j(\omega)}{\partial s_{p1}} E\rho_j(\hat{s}_{j2}(\omega), \hat{r}_{j2}(\omega)) d\omega \\
 &\quad + [U_j(\bar{\omega})(2\beta - \hat{s}_{p1}) - U_j(\underline{\omega})(2\alpha - \hat{s}_{p1})]\} \\
 &= 0
 \end{aligned}$$

$$(A.4) \quad \frac{\partial E\rho}{\partial \bar{r}_{\alpha 2}} = 0 \Rightarrow C(\bar{r}_{\alpha 2}) = R_{\alpha}$$

$$(A.5) \quad \frac{\partial E\rho}{\partial \bar{s}_{\alpha 2}} = 0 \Rightarrow \bar{s}_{\alpha 2} = 2\alpha$$

$$(A.6) \quad \frac{\partial E\rho}{\partial \bar{r}_{\beta 2}} = 0 \Rightarrow C(\bar{r}_{\beta 2}) = R_{\beta}$$

$$(A.7) \quad \frac{\partial E_p}{\partial \bar{s}_{\beta 2}} = 0 \Rightarrow \bar{s}_{\beta 2} = 2\beta$$

$$(A.8) \quad \frac{\partial E_p}{\partial r_{\alpha 2}(\omega)} = 0 \Rightarrow [R_{\alpha} - C(\hat{r}_{\alpha 2}(\omega))]$$

$$= \frac{1}{2} \lambda(\omega) \frac{s_{\beta}(\omega) N_{\beta} A_{\beta} (\hat{s}_{\alpha 2}(\omega))^{2\beta} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\beta}}{s_{\alpha}(\omega) N_{\alpha} A_{\alpha} (\hat{s}_{\alpha 2}(\omega))^{2\alpha} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\alpha}} [1 - F_{\beta}(\theta_{\beta}^*)]$$

$$(A.9) \quad \frac{\partial E_p}{\partial s_{\alpha 2}(\omega)} = 0 \Rightarrow [R_{\alpha} - \frac{1}{2} C(\hat{r}_{\alpha 2}(\omega))]$$

$$= \frac{1}{2} \lambda(\omega) \frac{s_{\beta}(\omega) N_{\beta} A_{\beta} (\hat{s}_{\alpha 2}(\omega))^{2\beta} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\beta}}{s_{\alpha}(\omega) N_{\alpha} A_{\alpha} (\hat{s}_{\alpha 2}(\omega))^{2\alpha} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\alpha}}$$

$$\times [1 - F_{\beta}(\theta_{\beta}^*)] \left[\frac{2\beta - \hat{s}_{\alpha 2}(\omega)}{2\alpha - \hat{s}_{\alpha 2}(\omega)} \right]$$

$$(A.10) \quad \frac{\partial E_p}{\partial r_{\beta 2}(\omega)} = 0 \Rightarrow [R_{\beta} - C(\hat{r}_{\beta 2}(\omega))]$$

$$= -\frac{1}{2} \lambda(\omega) [1 - F_{\beta}(\theta_{\beta}^*)]$$

$$(A.11) \quad \frac{\partial E_p}{\partial s_{\beta 2}(\omega)} = 0 \Rightarrow \hat{s}_{\beta 2}(\omega) = 2\beta$$

$$(A.12) \quad \frac{\partial E_p}{\partial \lambda(\omega)} = 0 \Rightarrow \frac{(\hat{s}_{\beta 2}(\omega))^{2\beta} (1 - \hat{s}_{\beta 2}(\omega))^{1-2\beta}}{(1 + \hat{r}_{\beta 2}(\omega))}$$

$$= \frac{(\hat{s}_{\beta 2}(\omega))^{2\beta} (1 - \hat{s}_{\beta 2}(\omega))^{1-2\beta}}{(1 + \hat{r}_{\beta 2}(\omega))}$$

Appendix B

Imperfect Strategy--Separate for the First Loan

Maximand

$$\begin{aligned}
 (B.1) \quad E\rho = & N_{\alpha} \{ E\rho_{\alpha}(s_{\alpha 1}, r_{\alpha 1}) + p_{\alpha} E\rho_{\alpha}(s_{\alpha 2}, r_{\alpha 2}) \\
 & + \int_{\Omega} g_{\alpha}(\omega) E\rho_{\alpha}(s_{\alpha 2}(\omega), r_{\alpha 2}(\omega)) d\omega \} \\
 & + N_{\beta} \{ E\rho_{\beta}(s_{\beta 1}, r_{\beta 1}) + p_{\beta} E\rho_{\beta}(s_{\beta 2}, r_{\beta 2}) + \lambda [E\pi_{\beta}(s_{\beta 1}, r_{\beta 1}) \\
 & + E\pi_{\beta}(s_{\beta 2}, r_{\beta 2}) - E\pi_{\beta}(s_{\alpha 1}, r_{\alpha 1}) - p_{\beta} \cdot 0 \\
 & - \int_{\Omega} g_{\beta}(\omega) E\pi_{\beta}(s_{\alpha 2}(\omega), r_{\alpha 2}(\omega)) d\omega] \}
 \end{aligned}$$

First Order Conditions

$$\begin{aligned}
 \text{(B.2)} \quad \frac{\partial E\rho}{\partial r_{\alpha 1}} = 0 &\Rightarrow N_{\alpha} \left\{ \frac{A_{\alpha} \hat{s}_{\alpha 1}^{2\alpha} (1 - \hat{s}_{\alpha 1})^{1-2\alpha}}{2(1 + \hat{r}_{\alpha 1})} [R_{\alpha} - C(\hat{r}_{\alpha 1})] \right. \\
 &- (1 + \hat{r}_{\alpha 1}) \int_{\Omega} \frac{\partial g_{\alpha}(\omega)}{\partial r_{\alpha 1}} E\rho_{\alpha}(\hat{s}_{\alpha 2}(\omega), \hat{r}_{\alpha 2}(\omega)) d\omega \\
 &+ [U_{\alpha}(\bar{\omega}) - U_{\alpha}(\underline{\omega})] \} \\
 &= \lambda N_{\beta} \{ E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) \\
 &- (1 + \hat{r}_{\alpha 1}) \int_{\Omega} \frac{\partial g_{\beta}(\omega)}{\partial r_{\alpha 1}} E\pi_{\beta}(\hat{s}_{\alpha 2}(\omega), \hat{r}_{\alpha 2}(\omega)) d\omega \\
 &+ [V_{\beta}(\bar{\omega}) - V_{\beta}(\underline{\omega})] \}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial E\rho}{\partial s_{\alpha 1}} = 0 &\Rightarrow N_{\alpha} \{ E\rho_{\alpha}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1})(2\alpha - \hat{s}_{\alpha 1}) \\
 (B.3) \quad &+ \hat{s}_{\alpha 1}(1 - \hat{s}_{\alpha 1}) \left[\frac{\partial p_{\alpha}}{\partial s_{\alpha 1}} E\rho_{\alpha}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2}) \right. \\
 &+ \left. \int_{Q_{\alpha 1}} \frac{\partial g_{\alpha}(\omega)}{\partial s} E\rho_{\alpha}(\hat{s}_{\alpha 2}(\omega), \hat{r}_{\alpha 2}(\omega)) d\omega \right] \\
 &+ [U_{\alpha}(\bar{\omega})(2\beta - \bar{s}_{\alpha 1}) - U_{\alpha}(\underline{\omega})(2\alpha - \bar{s}_{\alpha 1})] \} \\
 &= \lambda N_{\beta} \{ E\pi_{\beta}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1})(2\beta - \hat{s}_{\alpha 1}) \\
 &+ \hat{s}_{\alpha 1}(1 - \hat{s}_{\alpha 1}) \int_{Q_{\alpha 1}} \frac{\partial g_{\beta}(\omega)}{\partial s_{\alpha 1}} E\pi_{\beta}(\hat{s}_{\alpha 2}(\omega), \hat{r}_{\alpha 2}(\omega)) d\omega \\
 &+ [V_{\beta}(\bar{\omega})(2\beta - \hat{s}_{\alpha 1}) - V_{\beta}(\underline{\omega})(2\alpha - \hat{s}_{\alpha 1})] \}
 \end{aligned}$$

$$(B.4) \quad \frac{\partial E\rho}{\partial \bar{r}_{\alpha 2}} = 0 \Rightarrow C(\bar{r}_{\alpha 2}) = R_{\alpha}$$

$$(B.5) \quad \frac{\partial E\rho}{\partial \bar{s}_{\alpha 2}} = 0 \Rightarrow \bar{s}_{\alpha 2} = 2\alpha$$

$$(B.6) \quad \frac{\partial E\rho}{\partial r_{\alpha 2}(\omega)} = 0 \Rightarrow [R_{\alpha} - C(\hat{r}_{\alpha 2}(\omega))]$$

$$= \frac{\lambda}{2} \frac{g_{\beta}(\omega) N_{\beta} A_{\beta} (\hat{s}_{\alpha 2}(\omega))^{2\beta} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\beta}}{g_{\alpha}(\omega) N_{\alpha} A_{\alpha} (\hat{s}_{\alpha 2}(\omega))^{2\alpha} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\alpha}} [1 - F_{\beta}(\theta_{\beta}^*)]$$

$$\begin{aligned}
 (B.7) \quad \frac{\partial E\rho}{\partial s_{\alpha 2}(\omega)} = 0 &\Rightarrow [R_{\alpha} - \frac{1}{2} C(\hat{r}_{\alpha 2}(\omega))] \\
 &= \frac{\lambda}{2} \frac{s_{\beta}(\omega) N_{\beta}^A (\hat{s}_{\alpha 2}(\omega))^{2\beta} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\beta}}{s_{\alpha}(\omega) N_{\alpha}^A (\hat{s}_{\alpha 2}(\omega))^{2\alpha} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\alpha}} [1 - F_{\beta}(\theta_{\beta}^*)] \\
 &\quad \times \left[\frac{2\beta - \hat{s}_{\alpha 2}(\omega)}{2\alpha - \hat{s}_{\alpha 2}(\omega)} \right]
 \end{aligned}$$

$$(B.8) \quad \frac{\partial E\rho}{\partial r_{\beta 1}} = 0 \Rightarrow [R_{\beta} - C(\hat{r}_{\beta 1})] = -\frac{\lambda}{2} [1 - F_{\beta}(\theta_{\beta}^*)]$$

$$(B.9) \quad \frac{\partial E\rho}{\partial s_{\beta 1}} = 0 \Rightarrow \hat{s}_{\beta 1} = 2\beta$$

$$(B.10) \quad \frac{\partial F_{\beta}}{\partial r_{\beta 2}} = 0 \Rightarrow [R_{\beta} - C(\hat{r}_{\beta 2})] = -\frac{\lambda}{2} [1 - F_{\beta}(\theta_{\beta}^*)]$$

$$(B.11) \quad \frac{\partial E\rho}{\partial s_{\beta 2}} = 0 \Rightarrow \hat{s}_{\beta 2} = 2\beta$$

$$\begin{aligned}
 (B.12) \quad \frac{\partial E\rho}{\partial \lambda} = 0 &\Rightarrow \frac{\hat{s}_{\beta 1}^{2\beta} (1 - \hat{s}_{\beta 1})^{1-2\beta}}{(1 + \hat{r}_{\beta 1})} + \frac{\hat{s}_{\beta 2}^{2\beta} (1 - \hat{s}_{\beta 2})^{1-2\beta}}{(1 + \hat{r}_{\beta 2})} \\
 &= \frac{\hat{s}_{\alpha 1}^{2\beta} (1 - \hat{s}_{\alpha 1})^{1-2\beta}}{(1 + \hat{r}_{\alpha 1})} \\
 &\quad + \int_{\Omega} s_{\beta}(\omega) \frac{(\hat{s}_{\alpha 2}(\omega))^{2\beta} (1 - \hat{s}_{\alpha 2}(\omega))^{1-2\beta}}{(1 + \hat{r}_{\alpha 2}(\omega))} d\omega
 \end{aligned}$$

Introduction to Appendices C and D

The following notation is used throughout Appendices C and D:

$$E\rho_j(s_{jt}, r_{jt}) = p_j(s_{jt}) \left[\int_0^{1+r_{jt}} \theta_{jt} f_j(\theta_{jt}) d\theta_{jt} + \int_{1+r_j}^{\infty} (1 + r_{jt}) f_j(\theta_{jt}) d\theta_{jt} \right] - (1 + i)$$

$$E\pi_j(s_{jt}, r_{jt}) = p_j(s_{jt}) \int_{1+r_{jt}}^{\infty} [\theta_{jt} - (1 + r_{jt})] f_j(\theta_{jt}) d\theta_{jt}$$

$$R_j(r_{jt}) = \left[\int_0^{1+r_{jt}} \theta_{jt} f_j(\theta_{jt}) d\theta_{jt} + \int_{1+r_{jt}}^{\infty} (1 + r_{jt}) f_j(\theta_{jt}) d\theta_{jt} \right]$$

$$P_j(r_{jt}) = \int_{1+r_{jt}}^{\infty} [\theta_{jt} - (1 + r_{jt})] f_j(\theta_{jt}) d\theta_{jt}$$

Appendix C

Perfect Lender Strategy

Pool First Period; Separate Second Period

Second Period Maximand — Identified Firms: $j = \alpha, \beta$, $k = III, I$

$$\begin{aligned}
 (C.1) \quad E\rho^k = & \max\{p_j(s_{j2}^k) [\int_0^{1+r_{j2}^k} \theta_{j2} f_j(\theta_{j2}) d\theta_{j2} \\
 & + \int_{1+r_{j2}^k}^{\infty} (1 + r_{j2}^K) f_j(\theta_{j2}) d\theta_{j2}] - (1 + i) Q_{j2}^k \\
 & + \mu_{j2}^K [L_{j2}^K G_j(\gamma_{j2}^{*k}) - Q_{j2}^k] + \tilde{\mu}_{j2}^K [L_{j2}^K Q_{j1} - Q_{j2}^k]
 \end{aligned}$$

where

$$\gamma_{j2}^{*k} = p_j(s_{j2}^k) \int_{1+r_{j2}^k}^{\infty} [\theta_{j2} - (1 + r_{j2}^k)] f_j(\theta_{j2}) d\theta_{j2} .$$

First Order Conditions

$$(C.2) \quad \frac{\partial E\rho_k}{\partial s_{j2}} = 0 \quad \Rightarrow \quad \bar{s}_{j2}^k = j \quad .$$

$$(C.3) \quad \frac{\partial E\rho_k}{\partial r_{j2}} = 0 \quad \Rightarrow \quad Q_{j2}^k = \mu_{j2}^k L_j^k g_j(\gamma_{j2}^{*k})$$

$$(C.4) \quad \frac{\partial E\rho_k}{\partial Q_{j2}} = 0 \quad \Rightarrow \quad E\rho_j(\bar{s}_{j2}^k, \bar{r}_{j2}^k) = \mu_{j2}^k + \tilde{\mu}_{j2}^k \quad .$$

$$(C.5) \quad \frac{\partial E\rho_k}{\partial \mu_{j2}} = 0 \quad \Rightarrow \quad L_{j2}^k G_j(\gamma_{j2}^{*k}) = Q_{j2}^k \quad .$$

$$(C.6) \quad \frac{\partial E\rho_k}{\partial \mu_{j2}} \geq 0 \quad \Rightarrow \quad L_{j2}^k Q_{j1} \geq Q_{j2}^k \quad , \quad \tilde{\mu}_{j2}^k \geq 0 \quad , \quad \tilde{\mu}_{j2}^k [L_{j2}^k Q_{j1} - Q_{j2}^k] = 0$$

Second Period Maximand -- Unidentified Firms:

k = 0, II

$$\begin{aligned}
 (C.7) \quad E p^k = \max_{j=\alpha, \beta} \sum & \{ (p_j(s_{j2}^k) [\int_0^{1+r_{j2}^k} \theta_{j2} f_j(\theta_{j2}) d\theta_{j2} \\
 & + \int_{1+r_{j2}^k}^{\infty} (1 + r_{j2}^k) f_j(\theta_{j2}) d\theta_{j2}] - (1 + i) \} Q_{j2}^k [L_{jj}^k(\gamma_{j2}^{*k}) - Q_{j2}^k] \\
 & + \mu_{j2}^k [L_{jj}^k Q_{j1} - Q_{j2}^k] \} + \lambda \{ p_{\beta}(s_{\beta 2}^k) \int_{1+r_{\beta 2}^k}^{\infty} [\theta_{\beta 2} - (1 + r_{\beta 2}^k)] f_{\beta}(\theta_{\beta 2}) \\
 & - p_{\beta}(s_{\alpha 2}^k) \int_{1+r_{\alpha 2}^k}^{\infty} [\theta_{\beta 2} - (1 + r_{\alpha 2}^k)] f_{\beta}(\theta_{\beta 2}) d\theta_{\beta 2} \}
 \end{aligned}$$

where

$$\gamma_{j2}^{*k} = p_j(s_{j2}^k) \int_{1+r_{j2}^k}^{\infty} [\theta_{j2} - (1 + r_{j2}^k)] f_j(\theta_{j2}) d\theta_{j2} .$$

First Order Conditions

$$(C.8) \quad \frac{\partial E\rho^k}{\partial s_{\alpha 2}^k} = 0 \Rightarrow$$

$$\frac{p_{\alpha}^k(s_{\alpha 2}^k)}{p_{\beta}^k(s_{\alpha 2}^k)} = \frac{\lambda L_{\beta}^k P_{\beta}(r_{\alpha 2}^k)}{[R_{\alpha}(r_{\alpha 2}^k)Q_{\alpha 2}^k + \mu_{\alpha 2}^k L_{\alpha}^k g_{\alpha}(\gamma_{\alpha 2}^k)P_{\alpha}(r_{\alpha 2}^k)]}$$

$$(C.9) \quad \frac{\partial E\rho^k}{\partial r_{\alpha 2}^k} = 0 \Rightarrow$$

$$[Q_{\alpha 2}^k - \mu_{\alpha 2}^k L_{\alpha}^k g_{\alpha}(\gamma_{\alpha 2}^k)] = -\lambda L_{\beta}^k \frac{p_{\beta}(\hat{s}_{\alpha 2}^k)[1 - F_{\beta}(1 + \hat{r}_{\alpha 2}^k)]}{p_{\alpha}(\hat{s}_{\alpha 2}^k)[1 - F_{\alpha}(1 + \hat{r}_{\alpha 2}^k)]}$$

$$(C.10) \quad \frac{\partial E\rho^k}{\partial Q_{\alpha 2}^k} = 0 \Rightarrow E\rho_{\alpha}(s_{\alpha 2}^k, r_{\alpha 2}^k) = \mu_{\alpha 2}^k + \tilde{\mu}_{\alpha 2}^k$$

$$(C.11) \quad \frac{\partial E\rho^k}{\partial \mu_{\alpha 2}^k} = 0 \Rightarrow L_{\alpha}^k G_{\alpha}(\gamma_{\alpha 2}^k) = Q_{\alpha 2}^k$$

$$(C.12) \quad \frac{\partial E\rho^k}{\partial \tilde{\mu}_{\alpha 2}^k} = 0 \Rightarrow$$

$$L_{\alpha}^k Q_{\alpha 1}^k \geq Q_{\alpha 2}^k, \quad \tilde{\mu}_{\alpha 2}^k \geq 0, \quad \tilde{\mu}_{\alpha 2}^k [L_{\alpha}^k Q_{\alpha 1}^k - Q_{\alpha 2}^k] = 0.$$

$$(C.13) \quad \frac{\partial E\rho^k}{\partial s_{\beta 2}^k} = 0 \Rightarrow \hat{s}_{\beta 2}^k = \beta.$$

$$(C.14) \quad \frac{\partial E\rho^k}{\partial r_{\beta 2}^k} = 0 \Rightarrow [Q_{\beta 2}^k - \mu_{\beta 2}^k L_{\beta}^k g_{\beta}(\gamma_{\beta 2}^{*k})] = \lambda L_{\beta}^k .$$

$$(C.15) \quad \frac{\partial E\rho^k}{\partial Q_{\beta 2}^k} = 0 \Rightarrow E\rho_{\beta}(\hat{s}_{\beta 2}^k, \hat{r}_{\beta 2}^k) = \mu_{\beta 2}^k + \tilde{\mu}_{\beta 2}^k .$$

$$(C.16) \quad \frac{\partial E\rho^k}{\partial \mu_{\beta 2}^k} \geq 0 \Rightarrow$$

$$L_{\beta}^k G_{\beta}(\gamma_{\beta 2}^{*k}) \geq Q_{\beta 2}^k , \quad \mu_{\beta 2}^k \geq 0 , \quad \mu_{\beta 2}^k [L_{\beta}^k G_{\beta}(\gamma_{\beta 2}^{*k}) - Q_{\beta 2}^k] = 0 .$$

$$(C.17) \quad \frac{\partial E\rho^k}{\partial \mu_{\beta 2}^k} \geq 0 \Rightarrow L_{\beta}^k Q_{\beta 1} \geq 0 , \quad \tilde{\mu}_{\beta 2} \geq 0 , \quad \tilde{\mu}_{\beta 2}^k [L_{\beta}^k Q_{\beta 1} - Q_{\beta 2}^k] = 0 .$$

$$(C.18) \quad \frac{\partial E\rho^k}{\partial \lambda^k} = 0 \Rightarrow E\pi_{\beta}(\hat{s}_{\beta 2}^k, \hat{r}_{\beta 2}^k) = E\pi_{\beta}(\hat{s}_{\alpha 2}^k, \hat{r}_{\alpha 2}^k) .$$

First Period Maximand

(Assuming First-Period Contract Yields Higher Profit for Type β Firms than any Second-Period Contract)

$$\begin{aligned}
 \text{(C.19)} \quad E\rho = \max_{j=\alpha, \beta} & \left\{ p_j(s_{p1}) \left[\int_0^{1+r_{p1}} \theta_{j1} f_j(\theta_{j1}) d\theta_{j1} \right] \right. \\
 & + \int_{1+r_{p1}}^{\infty} (1+r_{p1}) f_j(\theta_{j1}) d\theta_{j1} - (1+i) \} Q_{j1} \\
 & + \mu_{j1} [G_j(\gamma_{j1}^*) - \theta_{j1}] \} + \sum_{k=0}^{III} E\rho^k
 \end{aligned}$$

where

$$\gamma_{j1}^* = p_j(s_{p1}) \int_{1+r_{p1}}^{\infty} [\theta_{j1} - (1+r_{p1})] f_j(\theta_{j1}) d\theta_{j1}, \quad j=\alpha, \beta.$$

First Order Conditions

$$(C.20) \quad \frac{\partial E\rho}{\partial s_{p1}} = 0 \Rightarrow$$

$$\frac{p'_\alpha(\hat{s}_{p1})}{p'_\beta(\hat{s}_{p1})} = \frac{-[R_\beta(\hat{r}_{p1})Q_{\beta 1} + \mu_{\beta 1}g_\beta(\gamma_{\beta 1}^*)P_\beta(\hat{r}_{p1}) + A_\beta]}{[R_\alpha(\hat{r}_{p1})Q_{\alpha 1} + \mu_{\alpha 1}g_\alpha(\gamma_{\alpha 1}^*)P_\alpha(\hat{r}_{p1}) + A_\alpha]} .$$

$$(C.21) \quad \frac{\partial E\rho}{\partial r_{p1}} = 0 \Rightarrow$$

$$\begin{aligned} & p_\alpha(\hat{s}_{p1})[1 - F_\alpha(1 + \hat{r}_{p1})]\{Q_{\alpha 1} - \mu_{\alpha 1}g_\alpha(\gamma_{\alpha 1}^*)\} \\ & = - p_\beta(\hat{s}_{p1})[1 - F_\beta(1 + \hat{r}_{p1})]\{Q_{\beta 1} - \mu_{\beta 1}g_\beta(\gamma_{\beta 1}^*)\} . \end{aligned}$$

$$(C.22) \quad \frac{\partial E\rho}{\partial Q_{\alpha 1}} = 0 \Rightarrow E\rho_\alpha(\hat{s}_{p1}, \hat{r}_{p1}) = \mu_{\alpha 1} .$$

$$(C.23) \quad \frac{\partial E\rho}{\partial Q_{\beta 1}} = 0 \Rightarrow E\rho_\beta(\hat{s}_{p1}, \hat{r}_{p1}) = \mu_{\beta 1} .$$

$$(C.24) \quad \frac{\partial E\rho}{\partial \mu_{\alpha 1}} = 0 \Rightarrow G(\gamma_{\alpha 1}^*) = Q_{\alpha 1} .$$

$$(C.25) \quad \frac{\partial E\rho}{\partial \mu_{\beta 1}} = 0 \Rightarrow G(\gamma_{\beta 1}^*) = Q_{\beta 1} .$$

where

$$A_{\alpha} = q_{\alpha} [\mu_{\alpha 2}^{III} G_{\alpha}(\gamma_{\alpha 2}^{*III}) + \tilde{\mu}_{\alpha 2}^{III} Q_{\alpha 1}] + (1 - q_{\alpha}) [\mu_{\alpha 2}^{II} G_{\alpha}(\gamma_{\alpha 2}^{*II}) + \tilde{\mu}_{\alpha 2}^{II} Q_{\alpha 1}]$$

$$- [\mu_{\alpha 2}^0 G_{\alpha}(\gamma_{\alpha 2}^{*0}) + \tilde{\mu}_{\alpha 2}^0 Q_{\alpha 1}] \quad .$$

$$A_{\beta} = q_{\beta} [\mu_{\beta 2}^I G_{\beta}(\gamma_{\beta 2}^{*I}) + \tilde{\mu}_{\beta 2}^I Q_{\beta 1}] + (1 - q_{\beta}) [\mu_{\beta 2}^{II} G_{\beta}(\gamma_{\beta 2}^{*II}) + \tilde{\mu}_{\beta 2}^{II} Q_{\beta 1}]$$

$$- [\mu_{\beta 2}^0 G_{\beta}(\gamma_{\beta 2}^{*0}) + \tilde{\mu}_{\beta 2}^0 Q_{\beta 1}] \quad .$$

First Period Maximand

(Assuming Second-Period Region II Contract Yields
Highest Expected Profit for Type β Firms)

$$\begin{aligned}
 (C.26) \quad E\rho = \max_{j=\alpha, \beta} \sum \{p_j(s_{p1}) [\int_0^{1+r_{p1}} \theta_{j1} f_j(\theta_{j1}) d\theta_{j1} \\
 + \int_{1+r_{p1}}^{\infty} (1+r_{p1}) f_j(\theta_{j1}) d\theta_{j1}] - (1+i)Q_{j1} + \mu_{\alpha 1} [G_{\alpha}(\gamma_{\alpha 1}^*) - Q_{\alpha 1}] \\
 + \tilde{\mu}_{\beta 1} [G_{\beta}(\tilde{\gamma}_{\beta 1}^*) - Q_{\beta 1}] + \sum_{k=0}^{III} E\rho^k \} .
 \end{aligned}$$

where

$$\begin{aligned}
 \gamma_{\alpha 1}^* &= p_{\alpha}(s_{p1}) \int_{1+r_{p1}}^{\infty} [\theta_{\alpha 1} - (1+r_{p1})] f_{\alpha}(\theta_{\alpha 1}) d\theta_{\alpha 1} \\
 \tilde{\gamma}_{\beta 1}^* &= \frac{1}{1 + p_{\beta}(s_{p1})(1 - q_{\beta})} \{ p_{\beta}(s_{p1}) \int_{1+r_{p1}}^{\infty} [\theta_{\beta 1} - (1+r_{p1})] f_{\beta}(\theta_{\beta 1}) d\theta_{\beta 1} \\
 &\quad + p_{\beta}(s_{p1})(1 - q_{\beta}) p_{\beta}(s_{\beta 2}^{II}) \int_{1+r_{\beta 2}^{II}}^{\infty} [\theta_{\beta 2} - (1+r_{\beta 2}^{II})] f_{\beta}(\theta_{\beta 2}) d\theta_{\beta 2} \} .
 \end{aligned}$$

First Order Conditions

$$(C.27) \quad \frac{\partial E\rho}{\partial s_{p1}} = 0 \Rightarrow \frac{p'_\alpha(\hat{s}_{p1})}{p'_\beta(\hat{s}_{p1})}$$

$$= \frac{-\{R_\beta(\hat{r}_{p1})Q_{\beta 1} + \tilde{\mu}_{\beta 1}g_\beta(\gamma_{\beta 1}^*) \frac{\tilde{\gamma}_{\beta 1}^*}{p_\beta(\hat{s}_{p1})[1 + p_\beta(\hat{s}_{p1})(1 - q_\beta)]} + A_\beta\}}{\{R_\alpha(\hat{r}_{p1})Q_{\alpha 1} + \mu_{\alpha 1}g_\alpha(\gamma_{\alpha 1}^*)P_\alpha(\hat{r}_{p1}) + \mu_{\beta 1}g_\beta(\gamma_{\beta 1}^*)p_\beta(\hat{s}_{p1})(1 - q_\beta)P_\alpha(\hat{r}_{p1})C^{II} + A_\alpha\}}$$

$$(C.28) \quad \frac{\partial E\rho}{\partial r_{p1}} = 0 \Rightarrow$$

$$p_\alpha(\hat{s}_{p1})[1 - F_\alpha(1 + \hat{r}_{p1})]\{Q_{\alpha 1} - \mu_{\alpha 1}g_\alpha(\gamma_{\alpha 1}^*)\}$$

$$= -p_\beta(\hat{s}_{p1})[1 - F_\beta(1 + \hat{r}_{p1})]\{Q_{\beta 1} - \tilde{\mu}_{\beta 1}g_\beta(\tilde{\gamma}_{\beta 1}^*)[\frac{1 + p_\beta(\hat{s}_{p1})(1 - q_\beta)C^{II}}{1 + p_\beta(\hat{s}_{p1})(1 - q_\beta)}]\} .$$

$$(C.29) \quad \frac{\partial E\rho}{\partial Q_{\alpha 1}} = 0 \Rightarrow E\rho_\alpha(\hat{s}_{p1}, \hat{r}_{p1}) = \mu_{\alpha 1} .$$

$$(C.30) \quad \frac{\partial E\rho}{\partial Q_{\beta 1}} = 0 \Rightarrow E\rho_\beta(\hat{s}_{p1}, \hat{r}_{p1}) = \tilde{\mu}_{\beta 1} .$$

$$(C.31) \quad \frac{\partial E\rho}{\partial \mu_{\alpha 1}} = 0 \Rightarrow G_\alpha(\gamma_{\alpha 1}^*) = Q_{\alpha 1} .$$

$$(C.32) \quad \frac{\partial E\rho}{\partial \mu_{\beta 1}} = 0 \Rightarrow G_\beta(\gamma_{\beta 1}^*) = Q_{\beta 1} .$$

where

$$A_{\alpha} = q_{\alpha} [\mu_{\alpha 2}^{III} G_{\alpha}(\gamma_{\alpha 2}^{*III}) + \tilde{\mu}_{\alpha 2}^{III} Q_{\alpha 1}] + (1 - q_{\alpha}) [\mu_{\alpha 2}^{II} G_{\alpha}(\gamma_{\alpha 2}^{*II}) + \tilde{\mu}_{\alpha 2}^{II} Q_{\alpha 1}] \\ - [\mu_{\alpha 2}^0 G_{\alpha}(\gamma_{\alpha 2}^{*0}) + \tilde{\mu}_{\alpha 2}^0 Q_{\alpha 1}] \quad .$$

$$A_{\beta} = q_{\beta} [\mu_{\beta 2}^I G_{\beta}(\gamma_{\beta 2}^{*I}) + \tilde{\mu}_{\beta 2}^I Q_{\beta 1}] + (1 - q_{\beta}) [\mu_{\beta 2}^{II} G_{\beta}(\gamma_{\beta 2}^{*II}) + \tilde{\mu}_{\beta 2}^{II} Q_{\beta 1}] \\ - [\mu_{\beta 2}^0 G_{\beta}(\gamma_{\beta 2}^{*0}) + \tilde{\mu}_{\beta 2}^0 Q_{\beta 1}] \quad .$$

$$C^{II} = \frac{p'_{\beta}(\hat{s}_{\alpha 2}) + p_{\beta}(\hat{s}_{\alpha 2}) [1 - F_{\beta}(1 + \hat{r}_{\alpha 2})]}{p'_{\alpha}(\hat{s}_{\alpha 2}) + p_{\alpha}(\hat{s}_{\alpha 2}) [1 - F_{\alpha}(1 + \hat{r}_{\alpha 2})]} \quad .$$

Appendix D

Imperfect Lender Strategy -- Separate

(First-period project outcome identifies technological type.)

Maximand

$$\begin{aligned}
 (D.1) \quad E\rho = & \sum_{t=1}^2 \left\{ p_{\alpha}(s_{\alpha t}) \left[\int_0^{1+r_{\alpha t}} \theta_{\alpha t} f_{\alpha}(\theta_{\alpha t}) d\theta_{\alpha t} + \int_{1+r_{\alpha t}}^{\infty} (1+r_{\alpha t}) f_{\alpha}(\theta_{\alpha t}) d\theta_{\alpha t} \right] \right. \\
 & - (1+i) + [1 - p_{\alpha}(s_{\alpha t})] 2\varepsilon Q_{\alpha t} + \mu_{\alpha t} [G_{\alpha}(\gamma_{\alpha t}^*) - Q_{\alpha t}] \left. \right\} + \tilde{\mu}_{\alpha 2} [Q_{\alpha 1} - Q_{\alpha 2}] \\
 & + 2 \left\{ p_{\beta}(s_{\beta L}) \left[\int_0^{1+r_{\beta L}} \theta_{\beta L} f_{\beta}(\theta_{\beta L}) d\theta_{\beta L} + \int_{1+r_{\beta L}}^{\infty} (1+r_{\beta L}) f_{\beta}(\theta_{\beta L}) d\theta_{\beta L} \right] \right. \\
 & - (1-i) + [1 - p_{\beta}(s_{\beta L})] \varepsilon Q_{\beta L} + \mu_{\beta L} [G_{\beta}(\gamma_{\beta L}^*) - \theta_{\beta L}] \left. \right\} \\
 & + \left\{ p_{\beta}(s_{\beta H}) \left[\int_0^{1+r_{\beta H}} \theta_{\beta H} f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} + \int_{1+r_{\beta H}}^{\infty} (1+r_{\beta H}) f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} \right] - (1+i) \right. \\
 & + [1 - p_{\beta}(s_{\beta H})] \varepsilon Q_{\beta H} + \mu_{\beta H} [G_{\beta}(\gamma_{\beta H}^*) - G_{\beta}(\gamma_{\beta L}^*) - Q_{\beta H}] \\
 & + \lambda \left\{ p_{\beta}(s_{\beta H}) \int_{1+r_{\beta H}}^{\infty} [\theta_{\beta H} - (1+r_{\beta H})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} \right. \\
 & \left. - p_{\beta}(s_{\alpha 1}) \int_{1+r_{\alpha 1}}^{\infty} [\theta_{\beta H} - (1+r_{\alpha 1})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H} \right\} ,
 \end{aligned}$$

where

$$\gamma_{\alpha 1}^* = \frac{1}{2} \sum_{t=1}^2 p_{\alpha}(s_{\alpha t}) \int_{1+r_{\alpha t}}^{\infty} [\theta_{\alpha t} - (1 + r_{\alpha t})] f_{\alpha}(\theta_{\alpha t}) d\theta_{\alpha t}$$

$$\gamma_{\alpha 2}^* = p_{\alpha}(s_{\alpha 2}) \int_{1+r_{\alpha 2}}^{\infty} [\theta_{\alpha 2} - (1 + r_{\alpha 2})] f_{\alpha}(\theta_{\alpha 2}) d\theta_{\alpha 2}$$

$$\gamma_{\beta L}^* = 2p_{\beta}(s_{\beta L}) \int_{1+r_{\beta L}}^{\infty} [\theta_{\beta L} - (1 + r_{\beta L})] f_{\beta}(\theta_{\beta L}) d\theta_{\beta L}$$

$$- p_{\beta}(s_{\beta H}) \int_{1+r_{\beta H}}^{\infty} [\theta_{\beta H} - (1 + r_{\beta H})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H}$$

$$\gamma_{\beta H}^* = p_{\beta}(s_{\beta H}) \int_{1+r_{\beta H}}^{\infty} [\theta_{\beta H} - (1 + r_{\beta H})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H}$$

First-Order Conditions

$$(D.2) \quad \frac{\partial E\rho}{\partial s_{\alpha 1}} = 0 \Rightarrow$$

$$\frac{p'_{\alpha}(\hat{s}_{\alpha 1})}{p'_{\beta}(\hat{s}_{\alpha 1})} = \frac{\lambda p_{\beta}(\hat{r}_{\alpha 1})}{\{R_{\alpha}(\hat{r}_{\alpha 1}) - 2\epsilon\}Q_{\alpha 1} + \frac{1}{2}g_{\alpha}(\gamma_{\alpha 1}^*)p_{\alpha}(\hat{r}_{\alpha 1})}$$

$$(D.3) \quad \frac{\partial E\rho}{\partial r_{\alpha 1}} = 0 \Rightarrow$$

$$\{Q_{\alpha 1} - \frac{1}{2}\mu_{\alpha 1}g_{\alpha}(\gamma_{\alpha 1}^*)\} = -\lambda \frac{p_{\beta}(\hat{s}_{\alpha 1})[1 - F_{\beta}(1 + \hat{r}_{\alpha 1})]}{p_{\alpha}(\hat{s}_{\alpha 1})[1 - F_{\alpha}(1 + \hat{r}_{\alpha 1})]}$$

$$(D.4) \quad \frac{\partial E\rho}{\partial Q_{\alpha 1}} = 0 \Rightarrow E\rho_{\alpha}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + [1 - p_{\alpha}(\hat{s}_{\alpha 1})]2\epsilon = \mu_{\alpha 1} - \tilde{\mu}_{\alpha 2}$$

$$(D.5) \quad \frac{\partial E\rho}{\partial \mu_{\alpha 1}} = 0 \Rightarrow G_{\alpha}(\gamma_{\alpha 1}^*) = Q_{\alpha 1}$$

$$(D.6) \quad \frac{\partial E\rho}{\partial s_{\alpha 2}} = 0 \Rightarrow \bar{s}_{\alpha 2} = \alpha$$

$$(D.7) \quad \frac{\partial E\rho}{\partial r_{\alpha 2}} \leq 0 \Rightarrow Q_{\alpha 2} \leq \frac{1}{2}\mu_{\alpha 1}g_{\alpha}(\gamma_{\alpha 1}^*) + \mu_{\alpha 2}g_{\alpha}(\gamma_{\alpha 2}^*)$$

$$(D.8) \quad \frac{\partial E\rho}{\partial r_{\alpha 2}} = 0 \Rightarrow E\rho_{\alpha}(\bar{s}_{\alpha 2}, \bar{r}_{\alpha 2}) + [1 - p_{\alpha}(\bar{s}_{\alpha 2})]2\epsilon = \mu_{\alpha 2} + \tilde{\mu}_{\alpha 2}$$

$$(D.9) \quad \frac{\partial E\rho}{\partial \mu_{\alpha 2}} = 0 \Rightarrow G_{\alpha}(\gamma_{\alpha 2}^*) = Q_{\alpha 2}$$

$$(D.10) \quad \frac{\partial E\rho}{\partial \mu_{\alpha 2}} = 0 \Rightarrow Q_{\alpha 1} \geq Q_{\alpha 2}$$

$$(D.11) \quad \frac{\partial E\rho}{\partial s_{\beta L}} = 0 \Rightarrow \hat{s}_{\beta L} = \beta$$

$$(D.12) \quad \frac{\partial E\rho}{\partial r_{\beta L}} = 0 \Rightarrow Q_{\beta L} = (2\mu_{\beta L} - \mu_{\beta H})g_{\beta}(\gamma_{\beta L}^*)$$

$$(D.13) \quad \frac{\partial E\rho}{\partial Q_{\beta L}} = 0 \Rightarrow E\rho_{\beta}(\hat{s}_{\beta L}, \hat{r}_{\beta L}) + [1 - p_{\beta}(\hat{s}_{\beta L})]\epsilon = \mu_{\beta L}$$

$$(D.14) \quad \frac{\partial E\rho}{\partial \mu_{\beta L}} = 0 \Rightarrow G_{\beta}(\gamma_{\beta L}^*) = Q_{\beta L}$$

$$(D.15) \quad \frac{\partial E\rho}{\partial s_{\beta H}} = 0 \Rightarrow \hat{s}_{\beta H} = \beta$$

$$(D.16) \quad \frac{\partial E\rho}{\partial r_{\beta H}} = 0 \Rightarrow \{Q_{\beta H} - [\mu_{\beta H}g_{\beta}(\gamma_{\beta L}^*) - (2\mu_{\beta L} - \mu_{\beta H})g_{\beta}(\gamma_{\beta L}^*)]\} = \lambda$$

$$(D.17) \quad \frac{\partial E\rho}{\partial Q_{\beta H}} = 0 \Rightarrow E\rho_{\beta}(\hat{s}_{\beta H}, \hat{r}_{\beta H}) + [1 - p_{\beta}(\hat{s}_{\beta H})]\epsilon = \mu_{\beta H}$$

$$(D.18) \quad \frac{\partial E\rho}{\partial \mu_{\beta H}} = 0 \Rightarrow G_{\beta}(\gamma_{\beta H}^*) - G_{\beta}(\gamma_{\beta L}^*) = Q_{\beta H}$$

$$(D.19) \quad \frac{\partial E\rho}{\partial \lambda} = 0 \Rightarrow$$

$$p_{\beta}(\hat{s}_{\beta H}) \int_{1+\hat{r}_{\beta H}}^{\infty} [\theta_{\beta H} - (1 + \hat{r}_{\beta H})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H}$$

$$= p_{\beta}(\hat{s}_{\alpha 1}) \int_{1+\hat{r}_{\alpha 1}}^{\infty} [\theta_{\beta H} - (1 + \hat{r}_{\alpha 1})] f_{\beta}(\theta_{\beta H}) d\theta_{\beta H}$$

Appendix E

Separation on Reservation Wages

E.I No Separation on Type α Borrowers

The lender cannot profitably alter the separating contracts derived from maximization of equation (10.5) under imperfect strategies by discriminating among the type α firms. Such a scheme would necessarily be characterized by two policy offerings for type α borrowers. The first is a lucrative contract, $(\hat{s}_{\alpha H}, \hat{r}_{\alpha H})$, available in the first period but no loan in the second. Alternatively, a type α borrower could select a contract requiring a higher interest payment in return for a guaranteed loan in the second period. These contracts are denoted $(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1})$ and $(\hat{s}_{\alpha 2}, \hat{r}_{\alpha 2})$, respectively. Only type α entrepreneurs with high reservation wages in the interval $(\gamma_{\alpha L}^*, \gamma_{\alpha H}^*]$ would select the single-loan option. Those with reservation wages in the range $[0, \gamma_{\alpha L}^*]$ would select the alternative contract offerings. The marginal borrower in each group is defined by

$$(E.1) \quad \gamma_{\alpha H}^* = E\pi_{\alpha}(\hat{s}_{\alpha H}, \hat{r}_{\alpha H})$$

and

$$(E.2) \quad \gamma_{\alpha L}^* = E\pi_{\alpha}(\hat{s}_{\alpha 1}, \hat{r}_{\alpha 1}) + E\pi_{\alpha}(\hat{s}_{\alpha 2}, \hat{r}_{\alpha 2}) - E\pi_{\alpha}(\hat{s}_{\alpha H}, \hat{r}_{\alpha H}) ,$$

respectively.

These suggested alterations in the type α contract offerings require modifications of the incentive constraints. In order to obtain

truthful revelation from type β entrepreneurs with high reservation wages, the lender must meet the condition

$$(E.3) \quad E\pi_{\beta}(\hat{s}_{\beta H}, \hat{r}_{\beta H}) \geq E\pi_{\beta}(\hat{s}_{\alpha H}, \hat{r}_{\alpha H}) .$$

Satisfaction of this inequality is sufficient to separate all firms on the basis of technological characteristics. The reservation wages of the type β entrepreneurs who select the two-period option are low enough to ensure

$$(E.4) \quad 2E\pi_{\beta}(s_{\beta L}, r_{\beta L}) \geq E\pi_{\beta}(s_{\beta H}, r_{\beta H}) + \gamma_{\beta} .$$

Consequently, if condition (E.3) holds, these borrowers will not be tempted to apply for any type α loans.

Consider the effects on the lender's expected returns induced by the proposed policy changes. The returns generated by the loans extended in the first period to type α entrepreneurs with high opportunity costs must cause the lender's share to diminish relative to the original contracts. However, the lender does gain from those type α borrowers with low reservation wages. Nevertheless, the overall expected returns from the type α firms must decrease in the first period. To understand this result, define

$$(E.5) \quad \xi = \frac{G_{\alpha}(\gamma_{\alpha H}^*) - G_{\alpha}(\gamma_{\alpha L}^*)}{G_{\alpha}(\gamma_{\alpha H}^*)} .$$

By assumption, the lender's return is a concave function of its arguments. Therefore, using Jensen's inequality and the assumption that

$E p_j(s, r)$ is concave, it can be concluded that a single type α contract derived as a convex combination of the two proposed first-period type α contracts yields at least as high a return to the lender:

$$(E.6) \quad \xi E p_{\alpha}(s_{\alpha H}, r_{\alpha H}) + (1 - \xi) E p_{\alpha}(s_{\alpha L}, r_{\alpha L}) \\ \leq E p_{\alpha}[\xi(s_{\alpha H}, r_{\alpha H}) + (1 - \xi)(s_{\alpha L}, r_{\alpha L})] .$$

The original first-period contract must be at least as profitable as the convex combination contract since it was derived as the profit-maximizing contract within the class of single type α offerings in the first period.

In the second period, the lender experiences further losses of expected returns from the loans extended to type α entrepreneurs. Originally, too few type α firms secured loans relative to the complete information case. Now the number of second-period type α borrowers has diminished further. Even if the lender charges a higher interest rate to these second-period borrowers, the total return from these loans must fall. The proposed change moves the contracts further from the complete information optimum. Therefore, it can be concluded that the lender derives no benefit from the proposed change with respect to the type α borrowers in the second period.

The lender's expected returns from type β borrowers must fall under the proposed contract changes. The first-period reward to those type β entrepreneurs with high reservation wages must rise to meet the incentive constraint, thereby reducing the lender's return from these

loans. The enhancement of the single-period type β contract requires an accompanying decrease in the interest rate charged to the two-period type β borrowers. To see this, rewrite equation (10.8) as

$$(E.7) \quad G_{\beta}(\gamma_{\beta 1}^*) - 2E_{\rho}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1})g_{\beta}(\gamma_{\beta 1}^*) = -E_{\rho}(\hat{s}_{\beta H}, \hat{r}_{\beta H})g_{\beta}(\gamma_{\beta 1}^*) .$$

The decrease in $\hat{r}_{\beta H}$ increases the RHS of equation (E.7). The LHS decreases in $\hat{r}_{\beta 1}$. Hence, $\hat{r}_{\beta 1}$ must rise with $\hat{r}_{\beta H}$. The lender must supplement the payoffs to each type β entrepreneur with a low opportunity cost. Therefore, the lender's expected returns from these firms must decrease as well. Hence, the lender must lose expected returns in the first period from all type β loans relative to the original contract offerings.

Within the type β technological category, only entrepreneurs with low opportunity costs apply for loans in the second period. As demonstrated above, the rewards for revelation of technological identity that are incorporated into these contracts must rise as a result of the proposed policy changes. Therefore, the lender's expected returns must fall from loans extended to type β firms in the second period.

The lender derives no gain in expected returns from loans extended to firms of either technological category in either period. Therefore, it can be concluded that the lender cannot profitably discriminate among the type α borrowers when following an imperfect separating strategy.

E.II. Separation on Type β Borrowers

The lender's optimal contract offerings when imperfection is allowed must be characterized by discrimination on the basis of

opportunity costs within the type β technological category. This will be demonstrated by contradiction.

Suppose instead that the lender offered only one type β contract in any given period. The contract for each period is denoted $(\hat{s}_{\beta 1}, \hat{r}_{\beta 1})$ and $(\hat{s}_{\beta 2}, \hat{r}_{\beta 2})$, respectively. It must be true that the parameters of these contracts be set such that

$$(E.8) \quad E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) > E\pi_{\beta}(\hat{s}_{\beta 2}, \hat{r}_{\beta 2}) .$$

The technology of each firm is identified by the first-period project return. Hence, the incentive constraint affects only the first-period contract. The reward to a type β firm for revealing its identity weakly satisfies inequality (E.8). The fact that the reduced first-period interest rate attracts more than the desired number of type β borrowers makes the inequality strict.

Those type β entrepreneurs who plan to borrow in both periods must have a reservation wage sufficiently low that

$$(E.9) \quad \gamma_{\beta} \leq \gamma_{\beta L}^* = E\pi_{\beta}(\hat{s}_{\beta 2}, \hat{r}_{\beta 2}) .$$

Define \tilde{r}_{β} as that interest rate which satisfies the condition

$$(E.10) \quad E\pi_{\beta}(\hat{s}_{\beta 1}, \hat{r}_{\beta 1}) + \gamma_{\beta L}^* = 2E\pi_{\beta}(\beta, \tilde{r}_{\beta}) .$$

All contracts offered to firms with type β technology are characterized by productive efficiency under a separating scheme; therefore, it follows from equation (E.10) that

$$(E.11) \quad \hat{r}_{\beta 1} < \tilde{r}_{\beta} < \hat{r}_{\beta 2} .$$

The lender could increase its expected returns by offering two sets of policy options that would discriminate among the type β borrowers on the basis of their reservation wages. The first option would include a first-period contract with the same terms as were proposed above, $(\hat{s}_{\beta 1}, \hat{r}_{\beta 1})$. A firm that chose this contract would be disqualified from any second-period loan. The other alternative available to type β entrepreneurs would be the contract $(\beta, \tilde{r}_{\beta})$, offered in each of the two periods.

All type β entrepreneurs with reservation wages below $\gamma_{\beta L}^*$ would choose the contract $(\beta, \tilde{r}_{\beta})$ in both periods. The lender's expected returns from these firms would rise relative to the contracts previously offered. This is due to the strict concavity of the lender's expected return function with respect to the interest rate. Since \tilde{r}_{β} can be represented as a convex combination of $\hat{r}_{\beta 1}$ and $\hat{r}_{\beta 2}$, it follows that

$$(E.12) \quad E_{\beta}(\hat{s}_{\beta 1}, r_{\beta 1}) + E_{\beta}(s_{\beta 2}, r_{\beta 2}) < 2E_{\beta}(\beta, \tilde{r}_{\beta}) .$$

Note that $E\pi_{\beta}(\beta, \tilde{r}_{\beta}) > \gamma_{\beta L}^*$, so all entrepreneurs within this group will secure loans in both periods.

All type β borrowers with reservation wages in the range $(\gamma_{\beta L}^*, \gamma_{\beta H}^*]$ choose the contract $(\hat{s}_{\beta 1}, \hat{r}_{\beta 1})$ in the first period and exit the market thereafter. The lender's total expected return from these firms is unaltered since they intended to borrow only once under the initial set of contract offerings.

Nothing has altered the type α contracts. Hence, no change in expected returns from these loans have been generated by the proposed policy.

The lender's expected returns over the two-period horizon are increased from the loans extended to the type β entrepreneurs with low opportunity costs under the proposed contract alterations. The returns from all other firms are unaltered. Therefore, it can be concluded that the lender's optimal set of contract offerings must be among the class of contracts characterized by discrimination within the type β technological category when imperfection is permitted.

Footnotes

- 1/ See Jaffee and Russell [1976], Bhattacharya [1980], and Koskela [1979].
- 2/ See Stiglitz and Weiss [1980]. For repeated play in markets other than credit markets, see Radner [1981] and Rubinstein and Yaari [1980].
- 3/ See, for example, Roosa [1951].
- 4/ See Jaffee and Modigliani [1968].
- 5/ See Stiglitz and Weiss [1980].
- 6/ See, for example, Wood [1975].
- 7/ See Jaffee and Modigliani [1969] and the accompanying comments.
- 8/ Moral hazard is not addressed in the dissertation model because the firms have no choice about their underlying technological types. However, since their contract selection does transmit some private information to the lender, problems of adverse selection do arise. The objective of the lender and every borrower is to maximize expected profit over the relevant time horizon; hence, all agents in the model are risk neutral by assumption. The issue of risk-sharing does not arise.
- 9/ Throughout the text, input and output variables chosen by the firms and contract terms chosen by the lender are double subscripted. The first subscript denotes the technology type (α , β , or p for pooling), and the second subscript distinguishes the first from the second loan (1 or 2). All such variables in the one-loan model carry the loan subscript 1 for consistency with later portions of the paper.
- 10/ This assumption is more than sufficient to guarantee that the lender's total expected return is non-negative. However, it will be convenient later.
- 11/ See Spence [1974].
- 12/ Separating contracts in a two-period problem are defined as contracts that insure unambiguous identification of technology types after the selection of the contract for the initial loan.

- 13/ This result was verified by means of a computer simulation study in a similar model that held the size of the loan fixed. These results are available from the author.
- 14/ Under the assumption of uniform distributions for the project returns generated by both types of technologies, both $g_{\alpha}(\omega)$ and $g_{\beta}(\omega)$ are constant. Therefore, the second-stage loan contracts are constant for all $\omega \in Q'$.
- 15/ $T_j(\bar{\omega})$ and $T_j(\underline{\omega})$ come from differentiation of $\bar{\omega}$ and $\underline{\omega}$, the endpoints of the subset Q' .
- 16/ If the technology of the two types of firms differed only in the probability of positive output, $p_j''(s) < 0$ would be sufficient to guarantee the desired regularity condition. The regularity condition is consistent with the form of the incentive problem assumed later in the text; namely, that the type β firms prefer the complete information type α contract to their own.
- 17/ At times the analysis will be aided by building more structure into the demand elasticities. A very simple way of achieving the desired structure is to assume that

$$G_j(\gamma_j) = g_j(\gamma_j)E\pi_j(s,r) .$$

In other words, the density of the reservation wage of potential borrowers is constant, and zero is the lower endpoint of the support. Since the actual distribution of reservation wages would be very difficult to estimate empirically, assuming a constant density seems fairly innocuous.

- 18/ The arguments of γ_j^* will be suppressed whenever the meaning is clear. The arguments are the parameters of the appropriate loan contract.
- 19/ Of course, lending institutions exist for more than a two-period horizon. New relationships are established with different borrowers at each stage. By assumption, the contractual relationships do not extend beyond two periods in this model. This is sufficient to capture the salient features of repeated negotiations. In this paper, only the transactions between the lender and a single cohort of borrowers are of interest. No "intergenerational effects" are studied.

- 20/ In each subsection of the two-loan model, a verbal description of the characteristics of the equilibrium will be presented before the formal analysis of the lender's maximization problem. The purpose of this "backward" procedure is to establish the binding incentive and quantity constraints facing the lender. This approach would not be necessary for the perfect separating strategy, but the intricacies of the models studied subsequently make the analysis formidable without first determining some preliminary results with regard to the constraints.
- 21/ The first subscript denotes the firm type. The second indicates the time period.
- 22/ Each second-period contract parameter in this subsection bears a subscript for technology type, a second subscript for time period, and a superscript denoting the region of the first-period outcome.

References

- Akerlof, G. [1970], "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," Quarterly Journal of Economics, Vol. 84, pp. 488-500.
- Bhattacharya, S. [1980]", "Nondissipative Signalling Structures and Dividend Policy," Quarterly Journal of Economics, Vol. 45, No. 1, pp. 1-24.
- "Billion Dollar Gamble, The" [1981], INC. Magazine, Vol. 3, No. 9.
- Dominguez, J. [1974], Performance and Prospects of the Venture Capital Industry, Lexington, Massachusetts: Lexington Books.
- Harris, M., and A. Raviv [1979], "Optimal Incentive Contracts with Imperfect Information," Journal of Economic Theory, Vol. 20, No. 2, pp. 231-259.
- Jaffee, D. and F. Modigliani [1969], "A Theory and Test of Credit Rationing," American Economic Review, Vol. 59, No. 5 pp. 850-72.
- Jaffee, D. and T. Russell [1976], "Imperfect Information, Uncertainty, and Credit Rationing," Quarterly Journal of Economics, Vol. 90, No. 4, pp., 651-666.
- Koskela, E. [1979], "On the Theory of Rationing Equilibria with Special Reference to Credit Markets: A Survey," Zeitschrift Fur Nationalokonomie, Vol. 39, No. 1-2, pp. 63-82.
- Marshall, M. [1981], "Time Is Ripe for Entrepreneurs," Electronics, Vol. 54, No. 17, pp. 93-104.
- Radner, Roy [1981], "Monitoring Cooperative Agreements in a Repeated Principal-Agent Relationship," Econometrica, Vol. 49, No. 5, pp. 1127-1148.
- Riley, J. [1979], "Informational Equilibrium," Econometrica, Vol. 47, No. 2, pp. 331-359.
- Roosa, R. [1951], "Interest Rates and the Central Bank," in Money, Trade and Economic Growth: Essays in Honor of John Henry Williams, Macmillan.
- Rogerson, W. [1982], "The Role of Reputation in a Repeated Agency Problem Involving Information Transmission," Technical Report No. 377, the Economics Series, Institute for Mathematical Studies in the Social Sciences, Stanford University.

- Rothschild, M., and J. Stiglitz [1976], "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics, Vol. 90, No. 4, pp. 629-649.
- Roussakis, E. [1977], Managing Commercial Bank Funds, New York: Praeger Publishers.
- Rubinstein, A., and M. Yaari [1980], "Repeated Insurance Contracts and Moral Hazard," mimeo, Hebrew University, Research Memorandum No. 37.
- Salop, J., and S. Salop [1976], "Self-Selection and Turnover in the Labor Market," Quarterly Journal of Economics, Vol. 90, No. 4, pp. 619-27.
- Shavell, S. [1979], "Risk Sharing and Incentives in Principal and Agent Problems," Bell Journal of Economics, Vol. 10, No. 1, pp. 55-73.
- Spence, M. [1974], Job Market Signaling, Cambridge: Harvard University Press.
- Stanhouse, B., and L. Sherman [1979], "A Note on Information in the Loan Evaluation Process," Journal of Finance, Vol. 34, No. 5, pp. 1236-1269.
- Stiglitz, J., and A. Weiss [1981], "Credit Rationing in Markets with Incomplete Information," American Economic Review, Vol. 71, No. 3, pp. 393-410.
- Stiglitz, J., and A. Weiss [1980], "Credit Rationing in Markets with Incomplete Information, Part II," mimeo, Bell Laboratories and Princeton University.
- "Venture Capital and the Media" [1981], Venture Capital Journal, Vol. 21, No. 9, p. 1.
- Wilson, C. [1977], "A Model of Insurance Markets with Incomplete Information," Journal of Economic Theory, Vol. 16, pp. 167-207.
- Wilson, C. [1980], "The Nature of Equilibrium in Markets with Adverse Selection," Bell Journal of Economics, Vol. 11, No. 1, pp. 108-30.
- Wood, J.H. [1975], Commercial Bank Loan and Investment Behavior, New York: John Wiley and Sons.

END

FILMED

2-86

DTIC